## Research Subject

## Dynamical Information Processing for Motion Pattern Generation and Transition (Nakamura Group)

## (1) Goal and summary

The information processing system for robots is designed so that it decides the robot motion based on the sensor signal. So far, robot motions are designed from the motion generation and motion decision points of view. The robot motions are represented by the symbols such as 'walk' or 'run', and the motion decision system is designed by a discrete system that obtains the optimal sequence of the symbols using the sensor signals. The motion generation system is designed as the robust controller on the assumption of the existence of the reference trajectory for the robot motion. In these methods, there are some problems as follows.

1. Because there is much information, it needs a large amount of calculation to process the information in the real world.
2. The possibility of the robot's motion and motion transition depends on the robustness of the controller.
3. The robot realizes only the pre-established motion.

On the other hand, Freeman[1, 2] shows the experimental result of the order for the known smell and the chaos for the unknown smell in the rabbit olfactory perception, and shows the close relationship between the nonlinear dynamical phenomenon and intelligence. Tsuda shows the effectiveness of the chaotic dynamics for learning and calls the phenomenon that the human brain transits some attractors as 'chaotic itinerancy' [3].

In this research, we design the dynamics-based information processing system for humanoid robots using dynamical phenomenon. The whole body motion of the humanoid robot is represented by a closed curve line in $N$ dimensional space, and we design a nonlinear dynamics that has a limit cyclic on this line.

The results of this research are listed as follows.

1. Since the humanoid robot has many degrees-of-freedom, it needs a large amount of calculation to deal with the whole body motion. In this research, we propose the reduction method for the humanoid robot's whole body motion using the correlation of the joint angle motion.
2. By proposing the design method of the nonlinear dynamics that has an attractor in $N$ dimensional space, we design the brain-like information processing system using the dynamics.
3. We design the dynamics-based information processing system that realizes the humanoid robot's smooth motion transition based on the sensor signal.
4. By the hierarchical configuration of the dynamical system, we design the continuous symbol space that decides the humanoid motion.

Whole body motion and dynamics In this paper, the time sequence data $Y$ that is obtained from the continuous time data $y(t)$ by the sampling time $T$

$$
Y=\left[\begin{array}{lll}
y(T) & y(2 T) & \cdots \tag{1}
\end{array}\right]
$$

is represented by

$$
Y=\left[\begin{array}{lll}
y[1] & y[2] & \cdots \tag{2}
\end{array}\right]
$$

Consider $\Xi \in \boldsymbol{R}^{N \times m}$ that consists of the time sequence data $\boldsymbol{\xi}[k] \in \boldsymbol{R}^{N}(k=1,2, \cdots)$ of the whole body motion $M$

$$
\begin{align*}
\Xi & =\left[\begin{array}{llll}
\boldsymbol{\xi}[1] & \boldsymbol{\xi}[2] & \cdots & \boldsymbol{\xi}[m]
\end{array}\right]  \tag{3}\\
\boldsymbol{\xi}[k] & =\left[\begin{array}{llll}
\xi_{1}[k] & \xi_{2}[k] & \cdots & \xi_{N}[k]
\end{array}\right]^{T} \tag{4}
\end{align*}
$$

where $m$ means the number of data. $\boldsymbol{\xi}[k]$ is assumed to satisfy

$$
\boldsymbol{\xi}[k+j m]=\boldsymbol{\xi}[k],\left\{\begin{array}{l}
k=1,2, \cdots m  \tag{5}\\
j=1,2, \cdots
\end{array}\right.
$$

and the motion $M$ is assumed to be a cyclic motion with period $m T$, then $M$ is represented by the closed curve line $C$ in $N$ dimensional space.

Let me consider the discrete time dynamics $\mathcal{D}$

$$
\begin{align*}
& \mathcal{D}: \boldsymbol{x}[k+1]=\boldsymbol{x}[k]+\boldsymbol{f}(\boldsymbol{x}[k])  \tag{6}\\
& \quad \boldsymbol{x}[k]=\left[\begin{array}{llll}
x_{1}[k] & x_{2}[k] & \cdots & x_{N}[k]
\end{array}\right]^{T} \in \boldsymbol{R}^{N} \tag{7}
\end{align*}
$$

The state vector $\boldsymbol{x}[k]$ means a point and moves in $N$ dimensional space. Assuming that the dynamics has an attractor on $C$, it memorizes the time sequence data and reproduces


Figure 1: Dynamics and whole body motion
the whole body motion.
Reduction of the whole body motion In the human whole body motion, the joint angles often have correlation with each other. For example, in the walk motion, joint angles of right body and left body are symmetrical. In this research, we propose two reduction methods.

## Motion reduction based on the nonlinear principal component analysis

Motion reduction based on the principal component analysis We propose the whole body motion reduction method using the principal component analysis. In this method, the motion data is decomposed into motion bases.

Consider the humanoid whole motion data represented by equation (3) and (4), where $N<m$ is assumed to be satisfied. When the matrix $\Xi \in \boldsymbol{R}^{N \times m}$ is represented as follows using $F$

$$
\begin{align*}
& \Xi=F X  \tag{8}\\
& F \in \boldsymbol{R}^{N \times r}, N>r  \tag{9}\\
& X=\left[\begin{array}{cccc}
x_{1}[1] & x_{1}[2] & \cdots & x_{1}[m] \\
\vdots & \vdots & & \vdots \\
x_{r}[1] & x_{r}[2] & \cdots & x_{r}[m]
\end{array}\right] \in \boldsymbol{R}^{r \times m} \tag{10}
\end{align*}
$$

we can consider that the $N$ dimensional matrix $\Xi$ is reduced to the $r$ dimensional matrix $X$. Though $F$ does not always exist, the approximate solution is obtained by the following singular value decomposition. Consider the singular value decomposition of $\Xi$.

$$
\begin{align*}
& \Xi=U S V^{T}  \tag{11}\\
& U \in \boldsymbol{R}^{N \times N}, S \in \boldsymbol{R}^{N \times m}, V \in \boldsymbol{R}^{m \times m} \tag{12}
\end{align*}
$$

where $U$ and $V$ are rectangular matrices, $S$ is the diagonal matrix whose elements are $\left\{\begin{array}{llll}s_{1} & s_{2} & \cdots & s_{N}\end{array}\right\}$ and the following is assume to be satisfied.

$$
\begin{equation*}
s_{1} \geq s_{2} \geq \cdots \geq s_{N} \geq 0 \tag{13}
\end{equation*}
$$

$s_{i}$ is the $i$-th singular value. By using $r$ singular values $s_{j}(1 \leq j \leq r<N)$ and singular vectors, we obtain the approximation of $\Xi$. By using the decomposition of

$$
\begin{align*}
& U=\left[\begin{array}{l|l}
U_{1} & U_{2}
\end{array}\right], \quad S=\left[\begin{array}{c|c}
S_{1} & 0 \\
\hline 0 & S_{2}
\end{array}\right], \quad V^{T}=\left[\begin{array}{c}
V_{1}^{T} \\
\hline V_{2}^{T}
\end{array}\right]  \tag{14}\\
& U_{1} \in \boldsymbol{R}^{N \times r}, \quad U_{2} \in \boldsymbol{R}^{N \times(N-r)}  \tag{15}\\
& S_{1} \in \boldsymbol{R}^{r \times r}, \quad S_{2} \in \boldsymbol{R}^{(N-r) \times(m-r)}  \tag{16}\\
& V_{1}^{T} \in \boldsymbol{R}^{r \times m}, \quad V_{2}^{T} \in \boldsymbol{R}^{(m-r) \times m} \tag{17}
\end{align*}
$$

$\Xi$ is approximated as follows.

$$
\begin{equation*}
\Xi \simeq U_{1} S_{1} V_{1}^{T} \tag{18}
\end{equation*}
$$

By writing $F$ and $X$ as

$$
\begin{align*}
& F=U_{1} S_{1}  \tag{19}\\
& X=V_{1}^{T} \tag{20}
\end{align*}
$$

we obtain the equation (8) and $\Xi$ is reduced to $r$ dimensional matrix $X$. From these results, we obtain the following consideration.

1. In equation (8), $m$ dimensional vectors

$$
\Xi_{i}=\left[\begin{array}{llll}
\xi_{i}[1] & \xi_{i}[2] & \cdots & \left.\xi_{i}[m]\right], \quad i=1,2, \cdots, N \tag{21}
\end{array}\right.
$$

is represented by $r$ vectors

$$
X_{i}=\left[\begin{array}{llll}
x_{i}[1] & x_{i}[2] & \cdots & \left.x_{i}[m]\right], \quad i=1,2, \cdots, r \tag{22}
\end{array}\right.
$$

Because the singular value decomposition in equation (11) is equivalent to the principal component analysis, the first vector $X_{1}$ is the first principal of the whole body motion $\Xi$.
2. By checking the singular value $s_{i}(i=1,2, \cdots, n)$, the appropriate $r$ is selected.
3. The first column vector of $V_{1}$ is the principal component of the motion $Y$, the second column vector is the second principal component.
4. The inverse function of $F$ is $S_{1}^{-1} U_{1}^{T}$

Design of the nonlinear dynamics that has an attractor on the closed curved line $C$ The nonlinear dynamics in equation (3) is designed so that that has an attractor on the closed curved line $C$. The design algorithm has the following 4 steps.


Figure 2: Time sequence data and closed curved line in $\xi$ space

Step1 As shown in Figure 2, the time sequence data of the cyclic whole body motion draws a closed curved line $C$ in the $N$ dimensional space. This line is assumed to not have a cross point.

Step2 Setting the domain $D$ in the $N$ dimensional space, define the vector field $\boldsymbol{f}\left(\boldsymbol{\eta}_{i}\right)$ on the points $\boldsymbol{\eta}_{i}$ in $D$ using $\gamma_{i}$ as shown in Figure 3.

$$
\begin{equation*}
\boldsymbol{f}\left(\boldsymbol{\eta}_{i}\right)=\left(\boldsymbol{\xi}^{\eta_{i}}[k+1]-\boldsymbol{\xi}^{\eta_{i}}[k]\right)+\boldsymbol{\gamma}_{i} \tag{23}
\end{equation*}
$$

where $\boldsymbol{\xi}^{\eta_{i}}[k]$ means the nearest point to $\boldsymbol{\eta}_{i}$ in $\boldsymbol{\xi}[k], \boldsymbol{\xi}^{\eta_{i}}[k+1]$ means $\boldsymbol{\xi}[k+1]$ that is the next time point of $\boldsymbol{\xi}^{\eta_{i}}[k]$. Define $\boldsymbol{\delta}_{i}[k]$ and $\boldsymbol{\delta}_{i}[k+1]$ as follows.

$$
\begin{align*}
\boldsymbol{\delta}_{i}[k] & =\boldsymbol{\eta}_{i}-\boldsymbol{\xi}^{\eta_{i}}[k]  \tag{24}\\
\boldsymbol{\delta}_{i}[k+1] & =\left(\boldsymbol{\eta}_{i}+\boldsymbol{f}\left(\boldsymbol{\eta}_{i}\right)\right)-\boldsymbol{\xi}^{\eta_{i}}[k+1] \tag{25}
\end{align*}
$$



Figure 3: Definition of the vector field

Let me explain about the decision of $\gamma_{i}$. When the state vector of the dynamics in equation (6) moves following the vector field $\boldsymbol{f}(\boldsymbol{x}[k])$, the sufficiency condition that the closed curved line $C$ becomes an attractor is

$$
\begin{equation*}
\left\|\boldsymbol{\delta}_{i}[k+1]\right\|<\left\|\boldsymbol{\delta}_{i}[k]\right\| \tag{26}
\end{equation*}
$$

and from equation (23), (24) and (25), we obtain

$$
\begin{equation*}
\boldsymbol{\delta}_{i}[k+1]=\boldsymbol{\delta}_{i}[k]+\boldsymbol{\gamma}_{i} \tag{27}
\end{equation*}
$$

From equation (26) we obtain

$$
\begin{equation*}
\left\|\boldsymbol{\delta}_{i}[k]+\boldsymbol{\gamma}_{i}\right\|<\left\|\boldsymbol{\delta}_{i}[k]\right\| \tag{28}
\end{equation*}
$$

that means that by selecting $\gamma_{i}$ satisfying equation (28), $\boldsymbol{\delta}_{i}[k]$ converges to 0 when $k \rightarrow \infty$.

Step 3 Set the some points $\boldsymbol{\eta}_{1}, \boldsymbol{\eta}_{2}, \cdots, \boldsymbol{\eta}_{L}$ in the domain $D$, define the vector $\boldsymbol{f}\left(\boldsymbol{\eta}_{1}\right)$, $\boldsymbol{f}\left(\boldsymbol{\eta}_{2}\right), \cdots, \boldsymbol{f}\left(\boldsymbol{\eta}_{L}\right)$ as shown in Figure 19.


Figure 4: Set of the vector field in the domain $D$

Step 4 The defined vector field $\boldsymbol{f}(\boldsymbol{x})$ is approximated by the $\ell$-th order polynomial of $\boldsymbol{x}$ as the following equation.

$$
\begin{equation*}
\boldsymbol{f}\left(\boldsymbol{\eta}_{i}\right)=\sum_{\substack{\ell=\emptyset_{1}, \cdots, p_{N} \\ p_{j} p_{j}=P}}^{\ell \sum_{\left(p_{1} p_{2} \cdots p_{N}\right)}^{\prod_{j=1}^{N}} \eta_{i j} p_{j}} \tag{29}
\end{equation*}
$$

where $a_{\left(p_{1} p_{2} \cdots p_{N}\right)}$ is the constant matrix. For example, in the case of $N=2$ and $\ell=3$, the polynomial function is represented by

$$
\begin{align*}
\boldsymbol{f}(\boldsymbol{x}) & =a_{(30)} x_{1}^{3}+a_{(21)} x_{1}^{2} x_{2}+a_{(12)} x_{1} x_{2}^{2}+a_{(03)} x_{2}^{3} \\
& +a_{(20)} x_{1}^{2}+a_{(11)} x_{1} x_{2}+a_{(02)} x_{2}^{2}+a_{(10)} x_{1}+a_{(01)} x_{2}+a_{(00)} \tag{30}
\end{align*}
$$

By representing $f(\boldsymbol{x})$ using $\Phi$ that consists of the coefficients of the polynomial function as follows,

$$
\begin{align*}
\boldsymbol{f}(\boldsymbol{x}) & =\Phi\left(a_{\left(p_{1} p_{2}\right.}^{\left.\cdots p_{N}\right)}\right)  \tag{31}\\
\boldsymbol{\theta}(\boldsymbol{x}) & =\left[\begin{array}{llllll}
x_{1}^{\ell} & \cdots & x_{N}^{\ell} & x_{1}^{\ell-1} x_{2} & \cdots & 1
\end{array}\right]^{T} \tag{32}
\end{align*}
$$

$\Phi$ is calculated by the least square method. By defining $F, \Theta$ as follows,

$$
\begin{align*}
& F=\left[\begin{array}{llll}
\boldsymbol{f}\left(\boldsymbol{\eta}_{1}\right) & \boldsymbol{f}\left(\boldsymbol{\eta}_{2}\right) & \cdots & \boldsymbol{f}\left(\boldsymbol{\eta}_{L}\right)
\end{array}\right]  \tag{33}\\
& \Theta
\end{align*}=\left[\begin{array}{llll}
\boldsymbol{\theta}\left(\boldsymbol{\eta}_{1}\right) & \boldsymbol{\theta}\left(\boldsymbol{\eta}_{2}\right) & \cdots & \boldsymbol{\theta}\left(\boldsymbol{\eta}_{L}\right) \tag{34}
\end{array}\right]
$$

$\Phi$ is obtained by the pseudo inverse matrix $\Theta^{\#}$,

$$
\begin{equation*}
\Phi\left(a_{\left(p_{1} p_{2} \cdots p_{N}\right)}\right)=F \Theta^{\#} \tag{35}
\end{equation*}
$$

and we obtain the nonlinear dynamics that has an attractor on $C$.

Figure 5 shows the motion of the designed dynamics in the case of $N=2$. " + " means the some initial value of the state vector $\boldsymbol{x}[0] . \boldsymbol{x}[k]$ converges to the closed curved line.

Next, we design the nonlinear dynamics that has some attractors. Because the nonlinear dynamics is designed based on the defined vector field in the domain $D$, it is able to have some attractors on the multi-closed curved line. However the more attractor is necessary to use the larger order of the polynomial. In this research, we represent the dynamics as the summation of the vector field.

Because the vector field $\boldsymbol{f}(\boldsymbol{x}[k])$ is defined only in the domain $D$, we set the effective domain $E$ of the approximated vector field using the ellipsoid that includes $D$ in the $N$ dimensional space. Consider the ellipsoid represented by the following equation.

$$
\begin{equation*}
\left(\boldsymbol{x}^{T}[k]-X_{0}^{T}\right) Q\left(\boldsymbol{x}[k]-X_{0}\right)=1 \tag{36}
\end{equation*}
$$

where $Q$ is positive definite, $X_{0}$ means the center of the ellipsoid. By using equation (36) and constant $a$, the weighting function $w(\boldsymbol{x}[k])$ is defined as follows.

$$
\begin{align*}
w(\boldsymbol{x}[k]) & =\frac{1}{1+\exp \{a(W(\boldsymbol{x}[k])-1)\}}  \tag{37}\\
W(\boldsymbol{x}[k]) & =\left(\boldsymbol{x}^{T}[k]-X_{0}^{T}\right) Q\left(\boldsymbol{x}[k]-X_{0}\right) \tag{38}
\end{align*}
$$



Figure 5: Motion of the nonlinear dynamics


Figure 6: The weighting function $w(x[k])$

The rough plot of this function is shown in Figure 6. When $\boldsymbol{x}[k]$ is inside the ellipsoid, $w(\boldsymbol{x}[k])$ is larger than 0 . By using $w(\boldsymbol{x}[k])$, we define the nonlinear dynamics $\mathcal{D}^{w}$ as follows.

$$
\begin{equation*}
\mathcal{D}^{w}: \boldsymbol{x}[k+1]=\boldsymbol{x}[k]+w(\boldsymbol{x}[k]) \boldsymbol{f}(\boldsymbol{x}[k]) \tag{39}
\end{equation*}
$$

The vector field $\boldsymbol{f}(\boldsymbol{x}[k])$ is effective only in the ellipsoid $E$ as shown in Figure 7. By using some dynamics $\mathcal{D}_{i}^{w}(i=1,2, \cdots)$, the nonlinear dynamics $\widetilde{\mathcal{D}}$ that has multi-attractors is designed as follows.

$$
\begin{align*}
\widetilde{\mathcal{D}}: \boldsymbol{x}[k+1] & =\boldsymbol{x}[k]+F(\boldsymbol{x}[k])  \tag{40}\\
F(\boldsymbol{x}[k]) & =\sum_{i} w_{i}(\boldsymbol{x}[k]) \boldsymbol{f}_{i}(\boldsymbol{x}[k]) \tag{41}
\end{align*}
$$

The multi-attractor dynamics is calculated by the low order polynomial function as shown in Figure 8. The multi-attractors (shown by $C_{i}$ ) are enclosed by the domain $E_{i}$.


Figure 7: Designed dynamics $\mathcal{D}^{w}$ using the weighting function


Figure 8: Multi-attractor dynamics $\widetilde{\mathcal{D}}$

Next, we design the humanoid whole body motion using the dynamics based information processing system. Figure 9 shows the humanoid robot (FUJITSU Humanoid HOAP-1) that has 20 degree of freedom. We design the "walk" motion and "squat" motion. Figure 10 shows the original motion. This humanoid robot is not grounded. Because the dynamic based information processing system yields only the time sequence of the joint angle, the feedback controller that stabilizes each motions should be implemented.

From the "walk" motion $\Xi_{w} \in \boldsymbol{R}^{20}$ and "squat" motion $\Xi_{s} \in \boldsymbol{R}^{20}$,

$$
\begin{align*}
\Xi_{w} & =\left[\begin{array}{llll}
\boldsymbol{\xi}_{w}[1] & \boldsymbol{\xi}_{w}[2] & \cdots & \boldsymbol{\xi}_{w}\left[m_{w}\right]
\end{array}\right]  \tag{42}\\
\Xi_{s} & =\left[\begin{array}{llll}
\boldsymbol{\xi}_{s}[1] & \boldsymbol{\xi}_{s}[2] & \cdots & \boldsymbol{\xi}_{s}\left[m_{s}\right]
\end{array}\right] \tag{43}
\end{align*}
$$

we obtain the reduced motions $X_{w} \in \boldsymbol{R}^{3}, X_{s} \in \boldsymbol{R}^{3}$ in three dimensional space.

$$
\begin{align*}
\Xi_{w} & =F_{w} X_{w}  \tag{44}\\
X_{w} & =\left[\begin{array}{llll}
\boldsymbol{x}_{w}[1] & \boldsymbol{x}_{w}[2] & \cdots & \boldsymbol{x}_{w}\left[m_{w}\right]
\end{array}\right]  \tag{45}\\
\Xi_{s} & =F_{s} X_{s}  \tag{46}\\
X_{s} & =\left[\begin{array}{llll}
\boldsymbol{x}_{s}[1] & \boldsymbol{x}_{s}[2] & \cdots & \boldsymbol{x}_{s}\left[m_{s}\right]
\end{array}\right] \tag{47}
\end{align*}
$$



Figure 9: Humanoid robot HOAP-1

Based on the reduced motion, we design the dynamics based on the following equation

$$
\begin{align*}
\boldsymbol{x}[k+1] & =\boldsymbol{x}[k]+\sum_{i=w, s} w_{i}(\boldsymbol{x}[k]) \boldsymbol{f}_{i}(\boldsymbol{x}[k])+\sum_{i=w, s} K_{i} O_{i}(\boldsymbol{x}[k])  \tag{48}\\
O_{i}(\boldsymbol{x}[k]) & =\delta\left(\boldsymbol{x}_{i}^{c}-\boldsymbol{x}[k]\right) \tag{49}
\end{align*}
$$

By changing $K_{w}$ and $K_{s}$, the humanoid transits its motion. $\delta$ is constant. $X_{w}^{c}$ and $X_{s}^{c}$ mean the center of reduced closed curved line "walk" and "squat" respectively. Figure 11 shows the motion of the dynamics. From the initial position, the dynamics is entrained to the walk motion (arrow 1), entrained to squat motion (arrow 2) and finally entrained to walk motion again (arrow 3).

Figure 12 shows the generated humanoid motion. Because while the dynamics is attracted to the closed curved line, the humanoid motion is as same as figure 10 , and only the transition motions are shown. The continuous transition is generated.

Motion transition based on the sensor signal We set the two space shown in Figure 13. One is the sensor space, another is motor space.

Sensor space : There multi-attractors in the sensor space. The state vector $\boldsymbol{x}_{s}[k]$ moves according to the vector field that is decided by the sensor signal.

Motor space : There are multi-attractors that define the humanoid's whole body motion. The state vector $\boldsymbol{x}_{m}[k]$ moves according to the vector field that is defined by the state vector $\boldsymbol{x}_{s}[k]$ in the sensor space.

For this configuration, we design the dynamics that has the input and output signals. The equation (40) is changed as follows.

$$
\begin{equation*}
\boldsymbol{x}[k+1]=\boldsymbol{x}[k]+w_{2}\left(w_{1} \boldsymbol{f}(\boldsymbol{x}[k])+\left(1-w_{1}\right) \delta\left(\boldsymbol{X}_{\boldsymbol{c}}-\boldsymbol{x}[k]\right)\right) \tag{50}
\end{equation*}
$$



Figure 10: Motion of the humanoid robot : "walk" and "squat"


Figure 11: Motion of the dynamics
where, $\delta(0<\delta<1)$ and $w_{2}$ are defined as follows by using $a_{2}$ and $\boldsymbol{X}_{c}$ that is the center of $C$.

$$
\begin{align*}
w_{2} & =1-\frac{1}{1+\exp \left\{a_{2}\left(\omega_{2}(\boldsymbol{x}[k])-1\right)\right\}}  \tag{51}\\
\omega_{2}(\boldsymbol{x}[k]) & =K\left(\boldsymbol{x}^{T}[k]-\boldsymbol{X}_{0}^{T}\right) Q\left(\boldsymbol{x}[k]-\boldsymbol{X}_{0}\right) \tag{52}
\end{align*}
$$

The parameter $K$ defines the basin of the attractor. Figure 14 shows the conceptual diagram. $E_{1}$ means the ellipsoid whose size is defined by $w_{1}$. The size of $E_{2}$ is defined by $K$. The nonlinear dynamics that has the input and output signal is represented as follows.

$$
\begin{equation*}
\boldsymbol{x}[k+1]=\boldsymbol{x}[k]+\sum_{i} w_{2 i}\left\{w_{1 i} \boldsymbol{f}_{i}(\boldsymbol{x}[k])+\left(1-w_{1 i}\right) \delta_{i}\left(\boldsymbol{X}_{c i}-\boldsymbol{x}[k]\right)\right\} \tag{53}
\end{equation*}
$$

The input is $K$ and the output is the number of the attractor that the state vector is entrained.

By using the proposed method, we generate the humanoid's whole body motion and motion transition. Figure 15 shows the upper body humanoid robot "Robovie". This robot has 11 joints, 16 on/off tactile sensors and CCD camera that detects the percentage of RGB from the image. We design 10 motions and reduce them in 3 dimension. Figure 16 shows the motion 1 and motion 2.

Based on equation (50), we design the dynamics in sensor space and motor space as follows.

$$
\begin{align*}
& \boldsymbol{x}^{s}[k+1]=\boldsymbol{x}^{s}[k]+\sum_{i=1}^{10} w_{2 i}^{m}\left\{w_{1 i}^{s} \boldsymbol{f}_{i}^{s}\left(\boldsymbol{x}^{s}[k]\right)+\left(1-w_{1 i}^{s}\right) \delta_{i}^{s}\left(\boldsymbol{X}_{c i}^{s}-\boldsymbol{x}^{s}[k]\right)\right\}  \tag{54}\\
& \boldsymbol{x}^{m}[k+1]=\boldsymbol{x}^{m}[k]+\sum_{i=1}^{10} w_{2 i}^{m}\left\{w_{1 i}^{m} \boldsymbol{f}_{i}^{m}\left(\boldsymbol{x}^{m}[k]\right)+\left(1-w_{1 i}^{m}\right) \delta_{i}^{m}\left(\boldsymbol{X}_{c i}^{m}-\boldsymbol{x}^{m}[k]\right)\right\} \tag{55}
\end{align*}
$$

Sensor signals

$$
\boldsymbol{u}_{s}=\left[\begin{array}{lllllll}
u_{s t 1} & u_{s t 2} & \cdots & u_{s t 16} & u_{s R} & u_{s G} & u_{s B} \tag{56}
\end{array}\right]^{T}
$$



Squat to walk
Figure 12: Motion transition of the humanoid robot


Figure 13: Hierarchical configuration of the dynamics using the sensor space and motor space


Figure 14: Change of basin


Figure 15: Upper body humanoid robot "Robovie"


Figure 16: Motion of the humanoid robot
are input that defines $K^{s}$

$$
K^{s}=\left[\begin{array}{llll}
K_{1}^{s} & K_{2}^{s} & \cdots & K_{10}^{s} \tag{57}
\end{array}\right]^{T}={ }^{s} W_{s} \boldsymbol{u}_{s}
$$

The entrainment to the attractor $\boldsymbol{w}_{s}$ of the dynamics in the sensor space

$$
\boldsymbol{w}_{s}=\left[\begin{array}{llll}
w_{1} & w_{2} & \cdots & u_{10} \tag{58}
\end{array}\right]^{T}
$$

defines the input $K^{m}$ to the motor space.

$$
K^{m}=\left[\begin{array}{llll}
K_{1}^{m} & K_{2}^{m} & \cdots & K_{10}^{m} \tag{59}
\end{array}\right]^{T}={ }^{m} W_{s} \boldsymbol{w}_{s}
$$

Figure 17 shows the trajectory of the dynamics in the motor space, where the sensor signal of the tactile sensors is changed right shoulder $\left(t_{1}<t<t_{2}\right) \rightarrow$, right of head $\left(t_{3}<t<t_{4}\right) \rightarrow$ and right $\operatorname{arm}\left(t_{5}<t\right)$. The experiment is held two times. The sequence of the sensor signal is same, but the timing is different, which causes the different robot's motion. Figure 18 shows the number of attractor that the state vector is entrained and number of the generated motion.


Figure 17: Trajectory of the dynamics in the motor space


Figure 18: The number of attractor that the state vector is entrained and generated motion

Design of continuous symbol space for humanoid motion Let me explain about the design of the symbol space and dynamics that handle the vector field in the motor space. The conceptual diagram is shown in Figure 19. One point in the symbol space defines the vector field in the motion space. According to the motion of the state vector of the dynamics in the symbol space, the vector field in the motion space is changed continuously that defines the motion generation and motion transition of the humanoid robot.


Figure 19: Symbol space and motion space
In the following, we will show the design method of the symbol space.
Step1 Consider the given two motions $M_{1}$ and $M_{2}$ represented by

$$
\begin{align*}
& M_{1}=\left[\begin{array}{llll}
\theta_{1}[1] & \theta_{1}[2] & \cdots & \theta_{1}[m] \\
M_{2} & =\left[\begin{array}{llll}
\theta_{2}[1] & \theta_{2}[2] & \cdots & \theta_{2}[m]
\end{array}\right]
\end{array} . \begin{array}{l}
\text { an }
\end{array}\right] \tag{60}
\end{align*}
$$

Step2 Obtain some motions between $M_{1}$ and $M_{2}$. For example, they are calculated by

$$
\begin{equation*}
M_{i}=\left(1-\alpha_{i}\right) M_{1}+\alpha_{i} M_{2}(i=3,4,5, \cdots) \tag{62}
\end{equation*}
$$

where $0<\alpha_{i}<1$ is satisfied.
Step3 Obtain $C_{i}$

$$
C_{i}=\left[\begin{array}{llll}
\boldsymbol{x}_{i}[1] & \boldsymbol{x}_{i}[2] & \cdots & \boldsymbol{x}_{i}[m] \tag{63}
\end{array}\right]
$$

that corresponds to $M_{i}$, where for all $i$ and $k, F(\cdot)$ that satisfies

$$
\begin{equation*}
\theta_{i}[k]=F\left(\boldsymbol{x}_{i}[k]\right) \tag{64}
\end{equation*}
$$

are assume to exist.
Step4 Design the nonlinear dynamics

$$
\begin{equation*}
\boldsymbol{\lambda}_{i}[k+1]=\boldsymbol{\lambda}_{i}[k]+\boldsymbol{g}_{i}\left(\boldsymbol{\lambda}_{i}[k]\right) \tag{65}
\end{equation*}
$$

that has an attractor on $C_{i} . \boldsymbol{g}_{i}\left(\boldsymbol{\lambda}_{i}[k]\right)$ is represented by

$$
\begin{equation*}
\boldsymbol{g}\left(\boldsymbol{\lambda}_{i}[k]\right)=\Phi_{i} \xi\left(\boldsymbol{\lambda}_{i}[k]\right) \tag{66}
\end{equation*}
$$

where $\Phi_{i}$ is the parameter matrix of the coefficient of the polynomial.

Step5 For some $\Phi_{i}$, obtain $\lambda$ and $\phi$ that satisfy

$$
\left[\begin{array}{c}
\Phi_{1}  \tag{67}\\
\Phi_{2} \\
\vdots \\
\Phi_{p}
\end{array}\right]=\left[\begin{array}{cccc}
\lambda_{11} I & \lambda_{12} I & \cdots & \lambda_{1 q} I \\
\lambda_{21} I & \lambda_{22} I & \cdots & \lambda_{2 q} I \\
\vdots & \vdots & & \vdots \\
\lambda_{p 1} I & \lambda_{p 2} I & \cdots & \lambda_{p q} I
\end{array}\right]\left[\begin{array}{c}
\phi_{1} \\
\vdots \\
\phi_{p}
\end{array}\right]
$$

Based on the singular value decomposition of $\Phi_{i}$

$$
\begin{gather*}
\Phi_{i}=\left[\begin{array}{lll}
\Phi_{1 i}^{T} & \cdots & \Phi_{N i}^{T}
\end{array}\right]^{T}  \tag{68}\\
\mathbf{\Phi}=\left[\begin{array}{cccc}
\Phi_{11} & \Phi_{21} & \cdots & \Phi_{N 1} \\
\Phi_{12} & \Phi_{22} & \cdots & \Phi_{N 2} \\
\vdots & \vdots & & \vdots \\
\Phi_{1 p} & \Phi_{2 p} & \cdots & \Phi_{N p}
\end{array}\right] \tag{69}
\end{gather*}
$$

$\lambda$ and $\phi$ are obtained by the following formulation.

$$
\boldsymbol{\Phi}=\left[\begin{array}{ccc}
\lambda_{11} & \cdots & \lambda_{1 q}  \tag{70}\\
\lambda_{21} & \cdots & \lambda_{2 q} \\
\vdots & & \vdots \\
\lambda_{p 1} & \cdots & \lambda_{p q}
\end{array}\right]\left[\begin{array}{c}
\phi_{1} \\
\vdots \\
\phi_{q}
\end{array}\right]
$$



Figure 20: Design of the motion space based on the symbol space

Step6 The vector $\boldsymbol{\lambda}_{i}$

$$
\boldsymbol{\lambda}_{i}=\left[\begin{array}{lll}
\lambda_{i 1} & \cdots & \lambda_{i q} \tag{71}
\end{array}\right]^{T}
$$

represents a point in the $q$ dimensional space, and $\boldsymbol{\lambda}_{i}$ defines the vector field in the motion space as follows.

$$
\Phi_{i}=\boldsymbol{\lambda}_{i}^{T}\left[\begin{array}{lll}
\phi_{1} & \cdots & \phi_{q} \tag{72}
\end{array}\right]^{T}
$$

This means that we design the continuous symbol space that defines the motion space and humanoid's motion.

The information process is shown in Figure 20. $\Phi$ is calculated by the summation of $\phi_{i}$ by the proportion of $\lambda_{i}$, and decides the motion of $\boldsymbol{x}[k]$.

We generate the humanoid motion using the proposed method. Consider the humanoid robot shown in Figure 9. We design the walk motion $W_{1}$ and squat motion $W_{2}$. By interpolating these motions, we design 22 motions that are reduced to 3 dimensional space obtaining the mapping function $F(\boldsymbol{x}[k])$. And the dynamics that has an attractor on each $C_{i}(i=1,2, \cdots, 22)$ respect to $M_{i}$ are designed as follows.

$$
\begin{equation*}
\boldsymbol{x}[k+1]=\boldsymbol{x}[k]+\Phi_{i} \xi(\boldsymbol{x}[k]) \tag{73}
\end{equation*}
$$

Based on the Step5, the 8 dimensional symbol space is designed. The generated motion is shown in Figure 21. In the symbol space, the state vector $\boldsymbol{x}_{s}[k]$ moves based on the dynamics entrained to the transition from the walk motion to squat motion. Only the 3















Figure 21: Motion generation based on the symbol space
axes are shown in symbol space. These figures show that the motion of the state vector in symbol space defines the motion of the state vector in the motion space that yields the humanoid's motion.

## (2) Results and their importance

In this research, we propose the design method of the dynamics-based information processing system. The results of this research are as follows.

1. Because the humanoid robot has many degrees-of-freedom, it requires large amount of computation to handle it. In this research, we propose the motion reduction method based on the nonlinear principal component analysis and principal component analysis. These methods make use of the correlation of the joint angle in the motion.
2. We show the design method of the nonlinear dynamics that has attractor on the closed curve line in the $N$ dimensional space, and proposed the dynamics-based information processing system that generates the humanoid motion.
3. By changing the basin of the attractor based on the sensor signal, we show the design method of the information processing system.
4. By the hierarchical configuration of the dynamics, we design the continuous symbol space and motion space that defines the humanoid motion and generates the continuous motion transition of the humanoid robot.

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