Variable Impedant Inverted Pendulum Model Control for a Seamless Contact Phase Transition on Humanoid Robot

Tomomichi Sugihara[1] and Yoshihiko Nakamura[1,2]

[1] University of Tokyo, 7–3–1 Hongo, Bunkyo-ku, Tokyo, 113-8656, Japan
sugihara@ynl.t.u-tokyo.ac.jp
[2] Japan Science and Technology Corporation(JST), CREST Program.

Abstract. Humanoid robots should be given much higher mobility, being expected as the utilities in the future. Especially, a controlling method which induces responsive motion from instantaneous decisions against disturbance should be emphasized on since real environment is filled with variations. This paper proposes Variable Impedant Inverted Pendulum model control to address a transition between contact and aerial phase to behave robustly against disturbance in the real environment, and to expand the range of their activities and perform a variety of motion. Manipulation of the external force is the key issue to enhance the mobility of humanoids since they are driven by the external force converted from the internal force through the interaction with the environment. VIIP model control which allows one to handle the external force rather easily. The advantage of proposed is that it is invariant on contact phase so that both cases in contact and in aerial are achieved in the unified way. The less amount of computation also helps realtime implementation. Then, Multiple Variable Impedant Imverted Pendulum(MVIIP) model is introduced, aiming at easy manipulation of contact state between each foot and the ground. We verified the effect of the controller in computer simulation, using a small humanoid robot model.

1 Introduction

The potential of humanoid robots as utilities in the future has not been exposed yet. In spite of having similar looks to human-beings, their present mobility are much lower than expected as machines which can act wherever humans can act. In order to put them into practical use in severe environment, they should be given more enhanced mobility. Although not a few studies in this field have improved the mobility of biped machines [1] [2] [3] [4] [5] [6], they have mainly addressed steady, periodic walking motion at all. Since real environment is filled with variations, a controlling method which induces responsive motion from instantaneous decisions against disturbance is required.

The authors[7] had proposed a COG(center of gravity) controlling method for humanoid robots through indirect manipulation of ZMP [1], which is the point of action of the total external force. It equivalently enables to handle horizontal components of the external force rather in a small amount of computation. Thanks to it, quick responsive motion and robust absorption of unpredicted impact in horizontal plane have been achieved. On the other hand, motion in vertical direction, which plays an important role for transition of contact phase, had not been enough taken into consideration. A transition of contact state is required i) to cope with emergency in the case that robots suffer from large impacts, and ii) to expand the range of their activities and perform a variety of motion.

In this paper, we augment it to realize a responsive motion transition between contact and aerial phase. The requirement for more positive manipulation of the external force is accomplished with Variable Impedant Inverted Pendulum(VIIP) model control. Some previous researches have achieved typical types of motion which go through aerial phase such as jumping and running with legged machines. Raibert et al.[8] realized hopping motion and even somersault with simple body robots by a combination of simple controlling rules. It owes to hydraulic actuator which works periodically, and is not a promising maneuver for humanoid robots. Nagasaka[9], Yamane et al.[10] and Kajita et al.[11] developed pattern planning methods of jumping and running for humanoids. Motion planning and control, however, are inseparable in nature since humanoids are dominated by non-holonomic constraints in accordance with the presence of underactuated links. Hirano et al.[12] studied jumping motion of a humanoid robot in computer simulation using an adaptive impedance control. It is for achievement of repetitive jumping, not responsive motion. Mita et al.[13] proposed Variable Constraint Control. Though it is effective against non-holonomic constraint, an explicit representation of equation of motion is needed. Thus, the more complicated the system is, the more amount of computation it requires. It is necessary for quick motion to make the amount of computation less and the control period short. Arikawa et al.[14] developed a Multi-DOF jumping Robot and controlled it according to pre-planned polynomial trajectories, which is not robust against disturbance. Pfeiffer et al.[15] are developing Jogging JOHNNIE, aiming at fast movement through running. It is still under development. Our stance is close to that of Mita et al. in the sense that manipulation of the external force is treated just as a part of the kinetic and dynamics constraints which denote the motion. Consequently, the controller is invariant against motion types, and even against contact phase. In addition, it has an advantage in small computational cost.

Then, we present the idea of Multiple Variable Impedant Inverted Pendulum(MVIIP) model control. VIIP is the model which puts an emphasis on COG control. Responsive legged motion, however, consists not only of COG control but of skillful footworks, namely, manipulation of reaction force and contact state of *each* foot. Not having any fixed point in the inertia frame, footworks of humanoid robots through manipulation of external reaction force converted from internal force(joint torques) often becomes a tough problem even in the case of a simple motion. Only a few researches have addressed this problem. Nishiwaki et al.[16] tried to generate stepping motion towards any direction composed of elements in motion database, pre-designed by genetic algorithm. Such a strategy doesn't explicitly take the conversion from internal force to external into account, so that it's hard to deal with a various type of terrain and to create a variety of motion with it. MVIIP is introduced, aiming at rather easy manipulation of contact state between feet and the ground. Impedance control are ordinarily adopted for the purpose of absorption of external force as disturbance. Park et al.[17] also applied it for the same purpose to walking motion. We focus on the aspect that it gives the system semi-passive characteristics which works to generate quick, reactive footworks.

2 Variable Inpedant Inverted Pendulum(VIIP) Model

2.1 VIIP Model

When in contact phase, the robot can convert the internal force generated at each joint actuator to the external reaction force through the interaction with the environment. VIIP model functions during this phase to control COG effectively.

Suppose z-axis coincides with vertical direction, $\boldsymbol{n}_Z$ is the total moment around ZMP, and $\boldsymbol{f}_G = [\, f_x \;\; f_y \;\; f_z \,]^T$, $\boldsymbol{n}_G = [\, n_x \;\; n_y \;\; n_z \,]^T$ are the force and moment acting at COG. In accordance with both the equation of motion and the equilibrium of moment, we get the following equations.

$$m(\ddot{\boldsymbol{p}}_G + \boldsymbol{g}) = \boldsymbol{f}_G \tag{1}$$

$$(\boldsymbol{p}_G - \boldsymbol{p}_Z) \times \boldsymbol{f}_G + \boldsymbol{n}_G = \boldsymbol{n}_Z \tag{2}$$

where $\boldsymbol{g} = [\, 0 \;\; 0 \;\; g \,]^T$ is the acceleration of gravity, and $\boldsymbol{p}_G = [\, x_G \;\; y_G \;\; z_G \,]^T$, $\boldsymbol{p}_Z = [\, x_Z \;\; y_Z \;\; z_Z \,]^T$ are the position of COG, ZMP respectively. Since the horizontal components of $\boldsymbol{n}_Z$ are zero by definition of ZMP,

$$\ddot{x}_G = \omega_G^2 (x_G - x_Z) - \frac{n_y}{m(z_G - z_Z)} \tag{3}$$

$$\ddot{y}_G = \omega_G^2 (y_G - y_Z) + \frac{n_x}{m(z_G - z_Z)} \tag{4}$$

$$\ddot{z}_G = \frac{f_z}{m} - g \tag{5}$$

where m is a total mass of the robot and ω_G is defined by

$$\omega_G^2 = \frac{f_z}{m(z_G - z_Z)} \tag{6}$$

One can associate Eq.(3)(4) with the dynamics of inverted pendulum although it has offset due to the existence of $\boldsymbol{n}_G$. Suppose the variation of $\boldsymbol{n}_G$ under the influence of the whole-body motion is so less than that of $\boldsymbol{f}_G$ that it can be neglected, $\boldsymbol{n}_G$ can be regarded as a constant value in a short term. This assumption is thought to be accepted in most cases of the legged locomotive motion. Consequently, the horizontal components of COG can be controlled through manipulation of ZMP as is already discussed in [7].

Here, we focus on Eq.(5), which denotes the COG motion in vertical direction. Since the robot is unfixed on the ground, f_z must satisfy the following constraint.

$$f_z \geq 0 \tag{7}$$

When f_z equals to zero, the robot is in aerial phase, while f_z is greater than zero, the robot is in contact. It means that manipulation of f_z plays an important role for the transition of contact phase. Detachment off the ground requires large f_z to accelerate COG enough against gravity. And, at the moment of touch down, f_z should be given compliant characteristic in order to absorb the impact. VIIP model shown in Fig.1 is that to realize such responsive and flexible motion in the unified way.

Based on the model, ${}^{ref}f_z$ is decided as

$${}^{ref}f_z = m\{K_{Pz}({}^{ref}z_G - z_G) + K_{Dz}({}^{ref}\dot{z}_G - \dot{z}_G) + g\} \tag{8}$$

where ${}^{ref}z_G$ is the referential COG in z-axis, whose meaning varies depending on the cases as is mentioned later. One can note that Eq.(8) includes a compensation of the gravity. Substituting this ${}^{ref}f_z$ for f_z in Eq.(5), we get

$$\ddot{z}_G = K_{Pz}({}^{ref}z_G - z_G) + K_{Dz}({}^{ref}\dot{z}_G - \dot{z}_G) \tag{9}$$

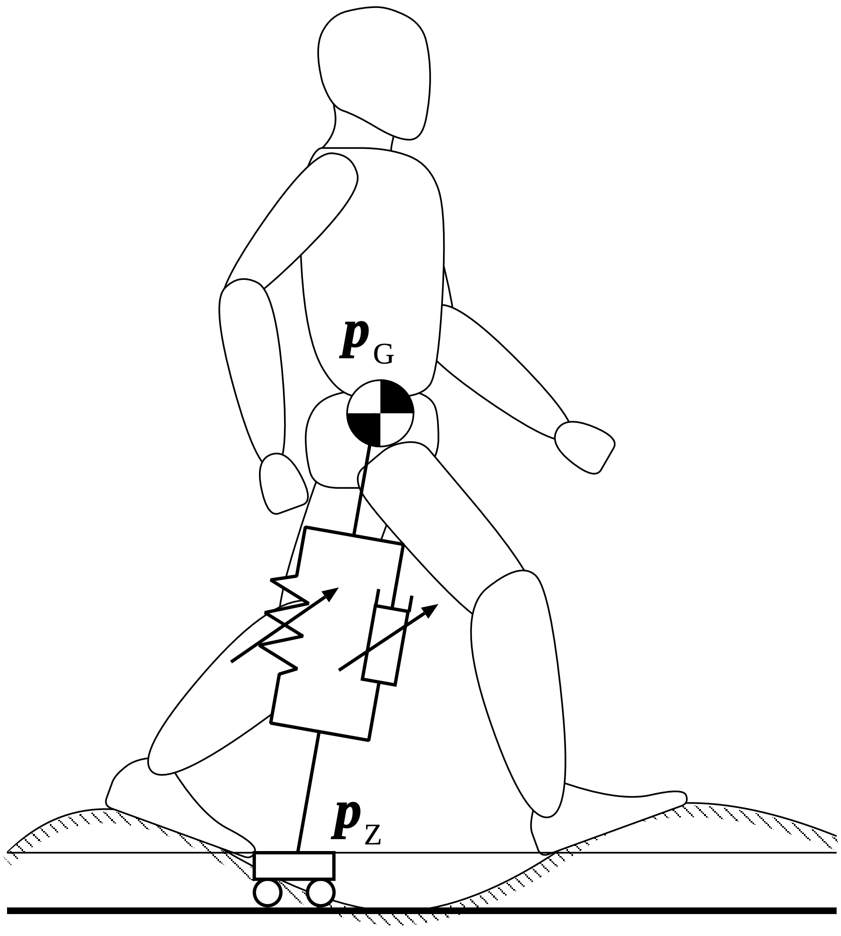


Fig. 1. Legged system and VIIP model

2.2 Design of Variable Impedance

K_{Pz} and K_{Dz} in Eq.(9) should be decided in accordance with the contact state and the motion scheme as is shown in Fig.2. I, II and III in Fig.2 are described as follows.

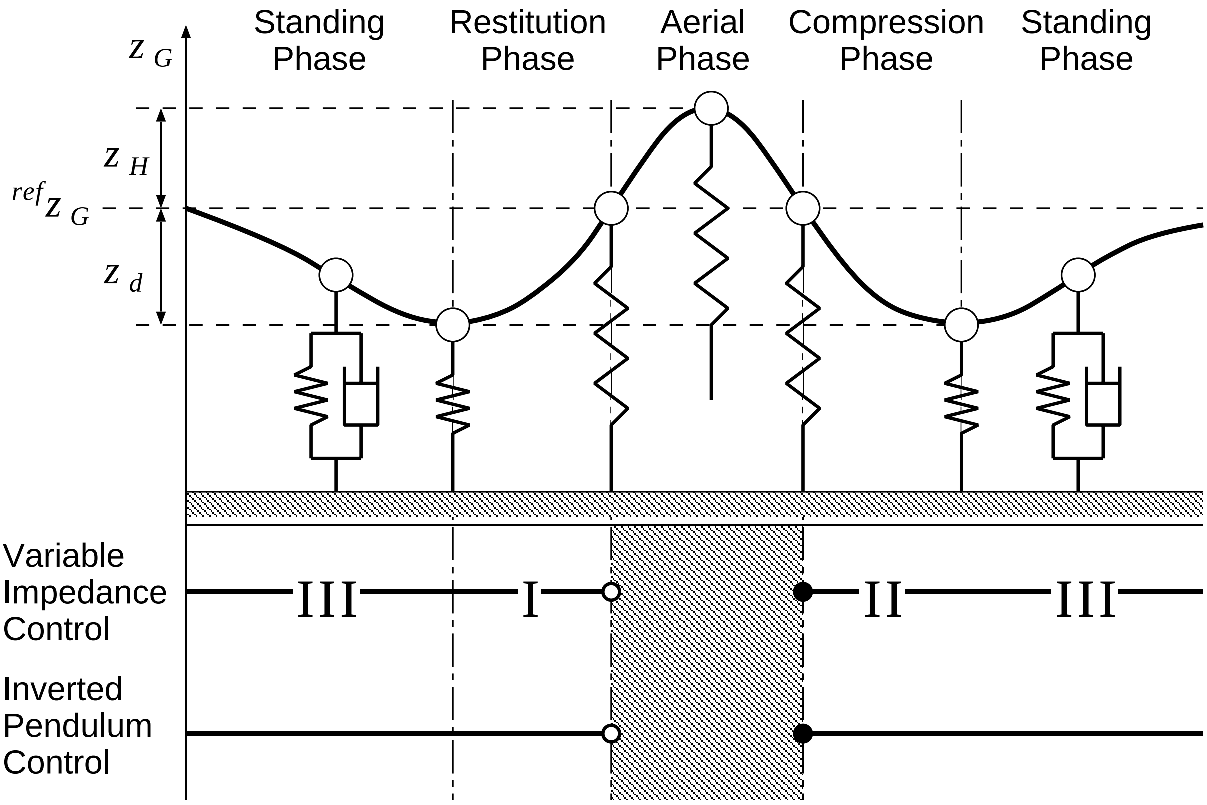


Fig. 2. Contact phase, impedance and inverted pendulum control

I)Impedance for lift-off

In addition to the acceleration of COG against gravity, velocity control is also essential to reach the desired height, since the initial velocity determines the maximum height in aerial phase. A simple spring model helps to meet this requirement. Suppose the robot lifts off when z_G equals to ${}^{ref}z_G$, the planned maximum jumping height from ${}^{ref}z_G$ in aerial phase is z_H, and stooping depth from ${}^{ref}z_G$ is z_d. In order to give the maximum vertical speed to the robot at the moment of detachment off, one should set K_{Dz} zero.

$$K_{Dz} = 0 \tag{10}$$

Then, we get the following equation from conservation of physical energy.

$$\frac{1}{2} m K_{Pz} z_d^2 = m g z_H \quad \Longleftrightarrow \quad K_{Pz} = \frac{2 g z_H}{z_d^2} \tag{11}$$

II)Impedance for touchdown

The spring model is also applicable for shock absorption at the touchdown. Suppose the robot lands onto the ground at the height ${}^{ref}z_G$ with falling speed immediately before contact $\dot{z}_{G-}$, and the desired maximum stooping depth from ${}^{ref}z_G$ is z_d. The impact at the touchdown is ideally eliminated if K_{Dz} is equal to zero. Then, we get

$$\frac{1}{2} m \dot{z}_{G-}^2 = \frac{1}{2} m K_{Pz} z_d^2 \quad \Longleftrightarrow \quad K_{Pz} = \left(\frac{\dot{z}_{G-}}{z_d} \right)^2 \tag{12}$$

which is also derived from conservation of physical energy.

III)Impedance in standing phase

Since Eq.(9) represents a second-order-lag system if both K_{Pz} and K_{Dz} are positive. Characteristic frequency and damping coefficient are

$$\omega = \sqrt{K_{Pz}}, \quad \zeta = \frac{K_{Dz}}{2\sqrt{K_{Pz}}} \tag{13}$$

For instance, z_G converges to ${}^{ref}z_G$ without overshoot when K_{Pz} and K_{Dz} satisfy the following condition.

$$\zeta > 1 \quad \Longleftrightarrow \quad K_{Dz}^2 - 4K_{Pz} > 0 \tag{14}$$

In standing phase, however, f_z must satisfy the following condition to remain contact with the ground.

$${}^{ref}f_z > 0 \tag{15}$$

Thus, one should limit ${}^{ref}f_z$ to an appropriate minimum value ${}^{ref}f_{z,min}(> 0)$.

2.3 Indirect Manipulation of External Reaction Force

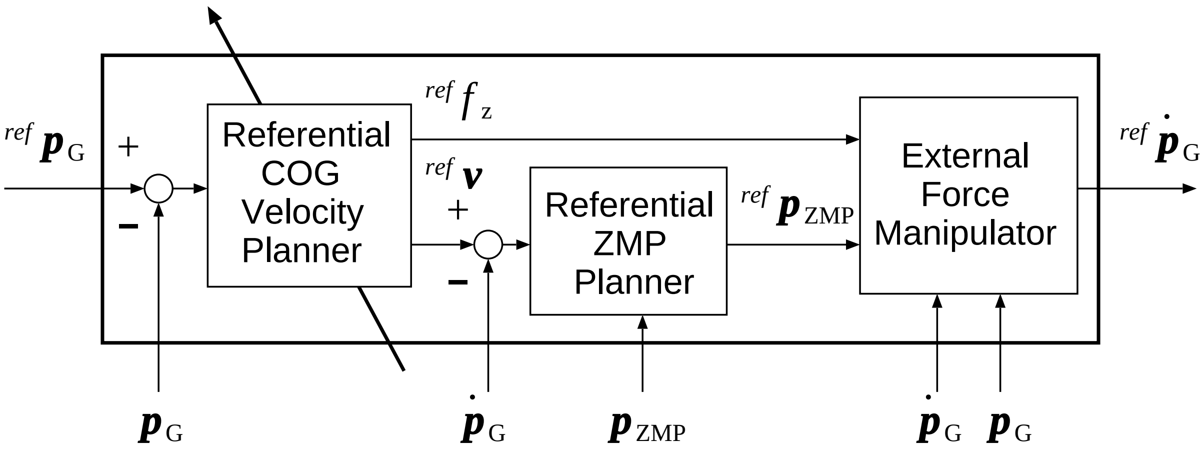


Fig. 3. The external force manipulator

Though the dynamics of legged system is similar to that of inverted pendulum, it is impossible to manipulate ZMP and vertical reaction force directly. Therefore, the equivalent internal force to manipulate them indirectly should be obtained.

When the acceleration ${}^{ref}\ddot{\boldsymbol{p}}_G$ in (16)(17)(18) is given to COG instantaneously, ZMP and vertical reaction force coincide with ${}^{ref}\boldsymbol{p}_Z$ and ${}^{ref}f_z$ respectively, in accordance with the equations (3)(4)(5).

$${}^{ref}\ddot{x}_G = {}^{ref}\omega_G^2(x_G - {}^{ref}x_Z) - \frac{n_y}{m(z_G - {}^{ref}z_Z)} \tag{16}$$

$${}^{ref}\ddot{y}_G = {}^{ref}\omega_G^2(y_G - {}^{ref}y_Z) + \frac{n_x}{m(z_G - {}^{ref}z_Z)} \tag{17}$$

$${}^{ref}\ddot{z}_G = \frac{{}^{ref}f_z}{m} - g \tag{18}$$

where ${}^{ref}\ddot{\boldsymbol{p}}_G = \begin{bmatrix} {}^{ref}\ddot{x}_G & {}^{ref}\ddot{y}_G & {}^{ref}\ddot{z}_G \end{bmatrix}^T$, and ${}^{ref}\omega_G$ is defined by

$${}^{ref}\omega_G^2 = \frac{{}^{ref}f_z}{m(z_G - {}^{ref}z_Z)} \tag{19}$$

and n_x, n_y are the current moment around x-axis, y-axis respectively. It requires a large amount of computation to calculate the equivalent internal force to the above acceleration exactly. Here, we integrate them and obtain the instantaneous strict referential velocity(SRV) of COG ${}^{ref}\dot{\boldsymbol{p}}_G$. COG Jacobian[7] $\boldsymbol{J}_G$ (refer to the appendix, too) can relate ${}^{ref}\dot{\boldsymbol{p}}_G$ to the referential motion of the whole joints ${}^{ref}\dot{\boldsymbol{\theta}}$ ($n\times 1$, n:number of the robot joints) as follows.

$$\boldsymbol{J}_G{}^{ref}\dot{\boldsymbol{\theta}} = {}^{ref}\dot{\boldsymbol{p}}_G \tag{20}$$

Fig.3 figures a block diagram of the manipulator. Inserting this equation as a constraint to the whole-body cooperative motion control described in the next section, a set of joint torque which approximately realizes ${}^{ref}\boldsymbol{f}_G$ is calculated.

Although it is a pragmatic approach, ignoring the inertial force other than the gravitation, it can significantly reduce the amount of computation so that it helps realtime implementation.

3 Contact Phase Invariant Whole-body Controller based on Constraints Switching and SRV Resolution

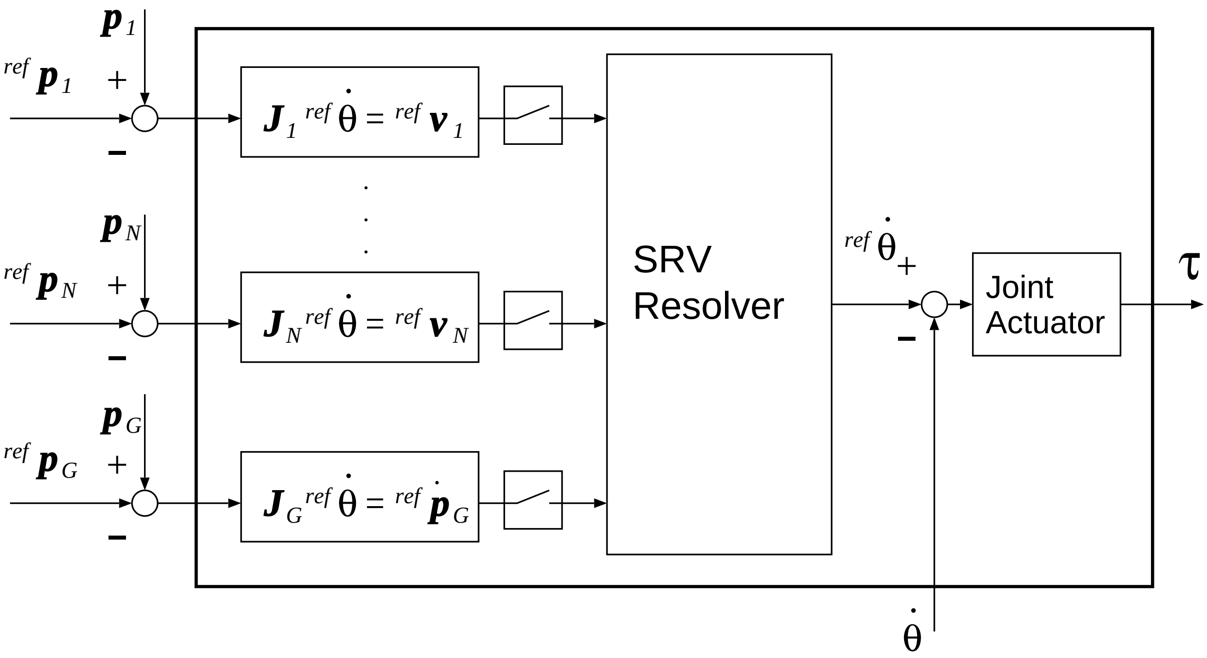


Fig. 4. Procedure of Constraints Swiching and SRV Resolution

Although the humanoid has tens of joints and thus is apparently complicated, the combination of various types of constraints due to contact with the environment and motion scheme itself determines a large part of the configuration of the robot. This idea has a similar aspect with what is so-called *synergetics* in biological field. Biological system in general consists of an extremely large number of muscles as active elements, so that the management of them seems quite challenging. However, natural ingenious mechanism – connection with bones and tendons, internal coupling of joints, contact with the environment, and so forth – functions as the constraint and reduces the actual degrees of freedom of the whole system. Then, the skillful whole-body cooperation is achieved as the result.

All constraints can be classified into those originated either in physical law or in controlling scheme. Suppose motion commands such as motion of arm or foot step are given by a set of strict referential velocity, represented in the following form.

$$\boldsymbol{J}\,{}^{ref}\dot{\boldsymbol{\theta}} = {}^{ref}\boldsymbol{v} \tag{21}$$

Eq.(21) can be regarded as a sort of constraint for control. In this sense, Eq.(20) is no more than a part of the constraints.

Physically achievable constraints must be selected in accordance with the contact condition. For instance, motion of the system is strongly constrained by conservation of momentum and angular momentum due to the absence of external input while in aerial phase. Therefore, COG is uncontrollable and

Eq.(20) is invalidated in this phase. The attitude and posture should be controlled instead, since they largely affects on the stability after landing as some former studies[8] revealed.

Now the problem is how to resolve ${}^{ref}\boldsymbol{v}$ into the whole body motion, namely, the motion of each joints. We translate it into the following quadratic programming to expect the minimum movement of each joint angle.

$$\begin{aligned} &\frac{1}{2}{}^{ref}\dot{\boldsymbol{\theta}}^{T}\boldsymbol{W}^{ref}\dot{\boldsymbol{\theta}} \longrightarrow \min. \\ &\text{subject to} \qquad \boldsymbol{J}^{ref}\dot{\boldsymbol{\theta}} = {}^{ref}\boldsymbol{v} \end{aligned} \tag{22}$$

where $\boldsymbol{W} \equiv diag\{w_i\}$ is the weighting matrix. And the solution is

$${}^{ref}\dot{\boldsymbol{\theta}} = \boldsymbol{W}^{-1}\boldsymbol{J}^{T}(\boldsymbol{J}\boldsymbol{W}^{-1}\boldsymbol{J}^{T})^{-1\,ref}\boldsymbol{v} \tag{23}$$

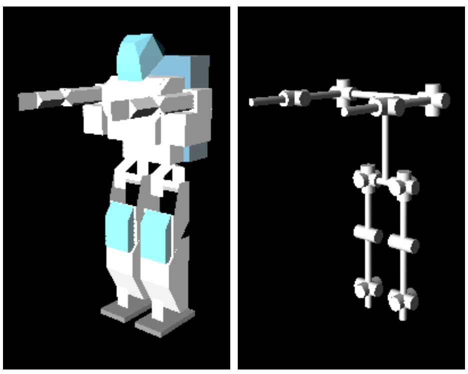
The set of joint torque which makes the joint angles follow ${}^{ref}\dot{\boldsymbol{\theta}}$ is calculated by local feedback controller, such as simple PD controller, at each joint.

Fig.4 shows the procedure, where

$$\boldsymbol{J}_i{}^{ref}\dot{\boldsymbol{\theta}} = {}^{ref}\boldsymbol{v}_i \tag{24}$$

is a partial constraint to denote the motion. They are switched in accordance with contact condition and motion scheme. In other words, the motion both in contact and in aerial phase is created only by switching of the constraints and no other procedure. Consequently, the controller stands invariable on contact phase. This is a similar idea to Variable Constraint Control[13] proposed by Mita et al., exept it denotes the constraints in a dimension of acceleration and force, whilc the method proposed in this paper denotes them in a dimension of velocity and momentum. This difference is attributable to the fact that we gave preference to the realtime implementation even in the case of humanoid robots as highly redundant system over the exactitude in the sense of dynamics.

4 Simulation



DOF: 20 (8 for arm,12 for leg)
height: 480 [mm]
weight: 6.5 [kg]

Fig. 5. Kinematic structure, size and mass of the robot

We realized a jumping motion in computer simulation with the controller proposed, using a robot model of HOAP-1[18](Fujitsu Automation Ltd.).

The planned maximum stooping depth, height at the detachment-off and the maximum height of jumping were 50[mm],220[mm] and 50[mm] respectively, and the referential height of COG after the touchdown was 220[mm]. Impedance in each phase were decided in accordance with values and equations derived in section 2.2 Fig.6 is a snapshot of the motion. And the loci of COG and ZMP are shown in Fig.7. We can see that the impedance control method works and the stable jumping motion is achieved.

5 Multiple Variable Inpedant Inverted Pendulum(MVIIP) Model Control

5.1 MVIIP Model

In this section, we introduce the idea of Multiple Variable Impedant Inverted Pendulum(MVIIP) model shown in Fig.8, aiming at easy manipulation of contact state between each foot and the ground.

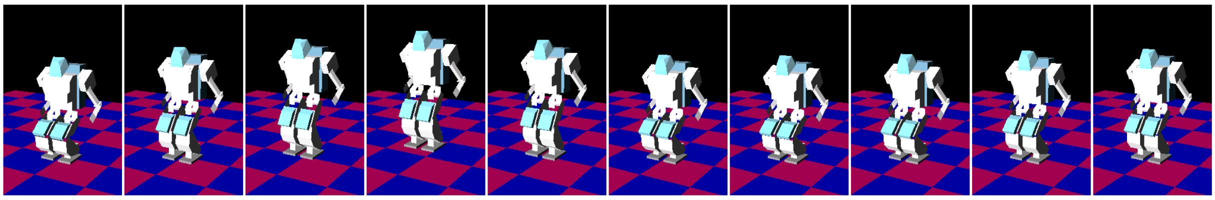


Fig. 6. Snapshot of a jumping motion simulation

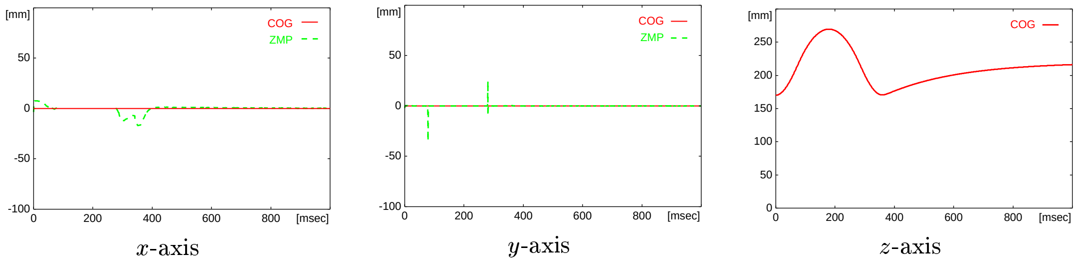


Fig. 7. Loci of COG and ZMP in each axis

Let us consider a simple case of stepping motion. At the moment of kicking against the ground, the reaction force acting to the foot of the kicking leg must be zero, that is to say, ZMP must completely be on the sole of the supporting leg. COG, however, should be accelerated enough in the direction towards the supporting leg in order to stay standing posture or go forward over the peak, which means, ZMP should be in the sole of the kicking leg at the beginning of kicking motion. As is shown by this fact, the implementation of footworks on humanoid robots requires a complicated manipulation of external reaction force and ZMP because of its underactuatedness and hyperstatic problem.

MVIIP model enables an intuitive manipulation of reaction force acting at each foot. ZMP is shifted in accordance with a distribution of them. MVIIP also gives the system semi-passive characteristics, which encourage quick, reactive footworks in addition to a compliant characteristic to absorb the impact as ordinary impedance control works.

5.2 Variable Design of Impedance

Here, we model the biped system just as shown in the left side of Fig.9. We also have the following assumptions.

1. The total mass is concentrated at the point $\boldsymbol{p}_G$.
2. Only telescopic force is applied at each leg.
3. Each foot contacts with the ground at one point, which is not connected to. Namely, when the force applied at the foot is 0, the foot detaches off the ground.
4. While in the noncontact phase, the foot position of the swinging leg is arbitrarily controllable.

We distinguish the two legs into the supporting leg S and the kicking leg K, and let f_X and $\boldsymbol{p}_X$ be the force applied at leg X and the foot position of leg X (X is for S or K) respectively.

Deciding f_K and f_S according to Eq.(25) and Eq.(26), the dynamics of the system becomes equivalent to one which consists of the supporting leg connected to the ground by one revolutional joint and the kicking leg having a spring-damper, just as shown in the right side of Fig.9.

$$f_K = \begin{cases} k({}^{ref}l_K - l_K) + c({}^{ref}\dot{l}_K - \dot{l}_K) & (l_K < {}^{ref}l_K) \\ 0 & (l_K \geq {}^{ref}l_K) \end{cases} \tag{25}$$

$$f_S = (m\boldsymbol{g} - f_K \boldsymbol{d}_K) \cdot \boldsymbol{d}_S \tag{26}$$

where

$$l_K = \|\boldsymbol{p}_G - \boldsymbol{p}_K\| \tag{27}$$

$$\boldsymbol{d}_K = (\boldsymbol{p}_G - \boldsymbol{p}_K)/l_K \tag{28}$$

$$\boldsymbol{d}_S = \frac{\boldsymbol{p}_G - \boldsymbol{p}_S}{\|\boldsymbol{p}_G - \boldsymbol{p}_S\|} \tag{29}$$

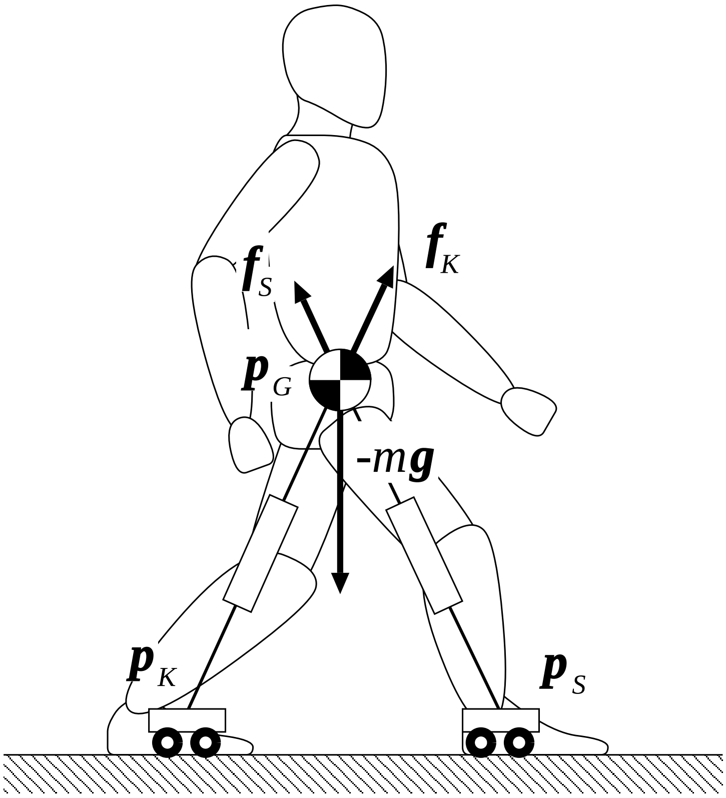


Fig. 8. Legged System and Multiple Variable Impedant Inverted Pendulum(MVIIP) model

and ${}^{ref}l_K$ is the virtual neutral length, which one can set arbitrarily, of the kicking leg. k and c are designed under the following rules.

I)Impedance against the gravity to kick the ground

In order to lift COG the height of h from the bottom of stooping(${}^{min}l_K$ the length of leg K at the point), k and c should be

$$c = 0, \quad k = \frac{2mgh}{({}^{ref}l_K - {}^{min}l_K)^2} \tag{30}$$

In this case, it is expected that leg K detaches off when its length becomes $l_K = {}^{ref}l_K$. If $h > l_S - {}^{min}h$ ($l_S = \| \boldsymbol{p}_G - \boldsymbol{p}_S \|$, and ${}^{min}h$ is the height of COT at the bottom of stooping), COG goes foward with the following speed v over the peak.

$$v = \sqrt{2g(h - l_S + {}^{min}h)} \tag{31}$$

II)Impedance for shock absorbing at the touchdown

In order to absorb the impact at the touchdown of the foot of leg K, k and c should be

$$c = 0, \quad k = \frac{mv_-^2}{({}^{ref}l_K - {}^{min}l_K)^2} \tag{32}$$

where ${}^{ref}l_K$ is the length of leg K at the touchdown, v_- is the COG speed in vertical direction immediately before touchdown and ${}^{min}l_K$ is the minimum length of leg K, which is at the bottom of stooping.

III)Impedance to stay standing

When k and c satisfy the following condition, COG converges to the referential position.

$$k > 0, \quad c > 0, \quad c^2 - 4k > 0 \tag{33}$$

Since f_K must satisfy the following condition to stay contact, $f_K \geq 0$ it should be limited to an appropriate minimum value ${}^{ref}f_{K,min}(> 0)$.

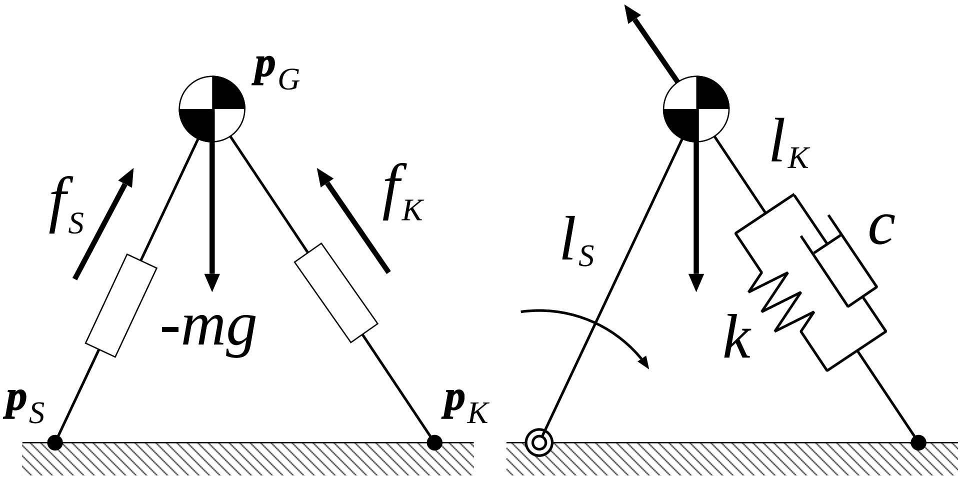


Fig. 9. Legged system and MVIIP model

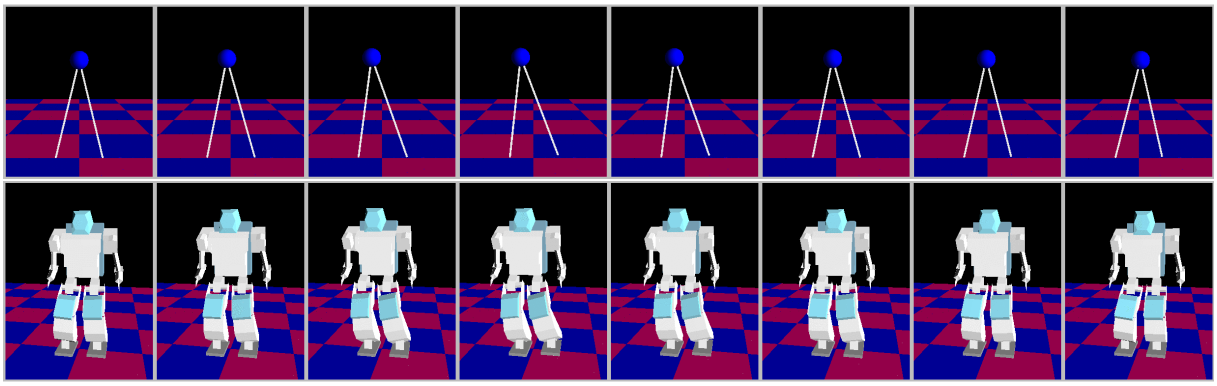


Fig. 10. Snapshots of a stepping motion realized by MVIIP model control

5.3 Indirect Manipulation of Reaction Force

Although a precise manipulation of reaction force at each foot is a difficult problem since the humanoid robot is a highly redundant multibody system with underactuated links, one can calculate the referential COG acceleration ${}^{ref}\ddot{\boldsymbol{p}}_G$ from the summation of referential $\boldsymbol{f}_K, \boldsymbol{f}_S$ and the gravity force equivallently as follows.

$$
{}^{ref}\ddot{\boldsymbol{p}}_G = \frac{\boldsymbol{f}_K + \boldsymbol{f}_S}{m} - \boldsymbol{g} \tag{34}
$$

Using Eq.(20), it turns to a constraint equation about COG.

When the both feet are in face contact, it becomes a hyperstatic problem; the points of action on each sole don't always coinside with desired $\boldsymbol{p}_S, \boldsymbol{p}_K$ even if the acceleration of COG is ${}^{ref}\ddot{\boldsymbol{p}}_G$. Though it didn't matter in the next simulation, it should be solved later, using such as robust force control.

We examined a stepping motion in a simulation based on proposed, letting the right leg be leg S and the left leg be leg K. Firstly, we set the initial stance between both feet was 10cm and the height of COG was 20cm. When incrementing the virtual neutral length of the supporting leg 2cm, a kicking motion immediately emerged and the left foot lifted off. After the left foot again touched down, we modified a damping coefficient, and COG converged to the position with a constant height. In this simulation, a PID control of the supporting leg to maintain its length was applied simultaneously with Eq.(26) to eliminate drift which was caused by the integral error. Fig.10 shows a snapshot of the motion of both a concentrated mass with weightless legs and HOAP-1 model created in the simulation.

6 Conclusion

We presented Variable Impedant Inverted Pendulum(VIIP) model, augmenting Inverted Pendulum Model Control in [7]. Thanks to it, indirect manipulation of external reaction force, especially the vertical component of the force is achieved rather easily, which enables the transition between contact and aerial phase with adequately designed impedance.

Handling of the external force, which generally requires so a large amount of computation that it is time-consuming, is translated into the COG velocity control equivalently, so that it has an advantage of less computational cost enough to be applied for realtime implementation.

Another advantage is that it is invariant on contact phase since the COG velocity control is expressed as no more than a part of the constraints which are switched in accordance with contact phase and motion scheme. The structure of controller itself doesn't vary in accordance with contact phase, and the motion both in contact and in aerial phase is realized in the unified way.

Acknowledgment

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Appendix: COG Jacobian

Since COG $\boldsymbol{p}_G$ is the function with an argument $\boldsymbol{\theta}$, there is a Jacobian $\boldsymbol{J}_G$ which relates $\dot{\boldsymbol{\theta}}$ to $\dot{\boldsymbol{p}}_G$.

$$\dot{\boldsymbol{p}}_G = \frac{\partial \boldsymbol{p}_G}{\partial \boldsymbol{\theta}}\dot{\boldsymbol{\theta}} \equiv \boldsymbol{J}_G\dot{\boldsymbol{\theta}} \tag{35}$$

We call this $\boldsymbol{J}_G$ *COG Jacobian*.

Hirano et al.[12] introduced the idea of COG Jacobian which were for a simple three-link model. In the case of practical humanoid robots, $\boldsymbol{J}_G$ is a quite complex non-linear function with multiple arguments. Tamiya et al.[19] proposed the method to calculate it using the numerical quasi-gradient, which needs a large amount of computation and also is less accurate. We developed a fast and accurate calculation method of $\boldsymbol{J}_G$ with the numerical approach as follows.

Firstly, The relative COG velocity with respect to the total body coordinate(which moves with the base link of the robot together) ${}^0\dot{\boldsymbol{p}}_G$ can be expressed as

$${}^0\dot{\boldsymbol{p}}_G = \frac{\sum_{i=0}^{n-1} m_i {}^0\dot{\boldsymbol{r}}_i}{\sum_{i=0}^{n-1} m_i} = \frac{\sum_{i=0}^{n-1} m_i {}^0\boldsymbol{J}_{Gi}\dot{\boldsymbol{\theta}}}{\sum_{i=0}^{n-1} m_i} \tag{36}$$

where m_i is the mass of link i, ${}^0\boldsymbol{r}_i$ is the position of the center of mass of link i with respect to the total body coordinate, and ${}^0\boldsymbol{J}_{Gi}$ ($3\times n$) is defined by

$${}^0\boldsymbol{J}_{Gi} \equiv \frac{\partial {}^0\boldsymbol{r}_i}{\partial \boldsymbol{\theta}} \tag{37}$$

${}^0\boldsymbol{J}_G$ is calculated easily by the method proposed by Orin et al.[20]

Therefore, Jacobian ${}^0\boldsymbol{J}_G$ which relates $\dot{\boldsymbol{\theta}}$ to ${}^0\dot{\boldsymbol{p}}_G$ is

$${}^0\boldsymbol{J}_G = \frac{\sum_{i=0}^{n-1} m_i {}^0\boldsymbol{J}_{Gi}}{\sum_{i=0}^{n-1} m_i} \tag{38}$$

Secondly, suppose link F is at rest in the inertia frame, as the foot link of the supporting leg for instance, the linear and angular velocity of the base link with respect to the world coordinate, $\dot{\boldsymbol{p}}_0$ and $\boldsymbol{\omega}_0$, are available from

$$\boldsymbol{\omega}_0 = -\boldsymbol{J}_{\omega F}\dot{\boldsymbol{\theta}} \tag{39}$$

$$\begin{aligned} \dot{\boldsymbol{p}}_0 &= -\boldsymbol{\omega}_0 \times \boldsymbol{R}_0 {}^0\boldsymbol{p}_F - \boldsymbol{R}_0 {}^0\dot{\boldsymbol{p}}_F \\ &= \boldsymbol{R}_0(-{}^0\boldsymbol{p}_F \times {}^0\boldsymbol{J}_{\omega F} - {}^0\boldsymbol{J}_F)\dot{\boldsymbol{\theta}} \end{aligned} \tag{40}$$

where $\boldsymbol{R}_0$ is the attitude matrix of the base link with respect to the world coordinate, ${}^0\boldsymbol{p}_F$ is the position of the link F in the total body coordinate, ${}^0\boldsymbol{\omega}_F$ is the relative rotation velocity of the link F with respect to the total body coordinate, ${}^0\boldsymbol{J}_F$ and ${}^0\boldsymbol{J}_{\omega F}$ are the Jacobian about relative linear and angular velocity of the link F with respect to the total body coordinate, and the notation $[\boldsymbol{v}^\times]$ means outer-product matrix of a vector $\boldsymbol{v}$ (3×1).

Then, the COG velocity with respect to the world coordinate $\dot{\boldsymbol{p}}_G$ is

$$\begin{aligned} \dot{\boldsymbol{p}}_G &= \dot{\boldsymbol{p}}_0 + \boldsymbol{\omega}_0 \times \boldsymbol{R}_0 {}^0\boldsymbol{p}_G + \boldsymbol{R}_0 {}^0\dot{\boldsymbol{p}}_G \\ &= \boldsymbol{R}_0\{{}^0\dot{\boldsymbol{p}}_G - {}^0\dot{\boldsymbol{p}}_F + ({}^0\boldsymbol{p}_G - {}^0\boldsymbol{p}_F) \times {}^0\boldsymbol{\omega}_F\} \\ &= \boldsymbol{R}_0\{{}^0\boldsymbol{J}_G - {}^0\boldsymbol{J}_F + [({}^0\boldsymbol{p}_G - {}^0\boldsymbol{p}_F)^\times]{}^0\boldsymbol{J}_{\omega F}\}\dot{\boldsymbol{\theta}} \end{aligned} \tag{41}$$

And we can conclude that $\boldsymbol{J}_G$ is

$$\boldsymbol{J}_G = \boldsymbol{R}_0\{{}^0\boldsymbol{J}_G - {}^0\boldsymbol{J}_F + [({}^0\boldsymbol{p}_G - {}^0\boldsymbol{p}_F)^\times]{}^0\boldsymbol{J}_{\omega F}\} \tag{42}$$