Development of a Biped Humanoid Simulator

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Abstract

Since a biped humanoid inherently suffers from instability and always risks to tipping over, stable and reliable biped walking is the most important goal. The simulator is a significant tool to pursue this goal. In this paper, we first present a method for constructing a humanoid simulator that can closely model and predict the motion of an actual humanoid. We then propose a balance controller consisting of an off-line walk-pattern generater and a real-time modification. Using the simulator, we can predict the humanoid's physical capability subject to the constraints of actuators, and clarify the required specifications of actuators to execute a desired task. The functions of the developed simulator and the effectiveness of the proposed balance controller were evaluated through simulated walks on an unknown rough terrain, soft ground, and an environment in the presence of disturbances.

1 Introduction

In aging societies, the need of robots to assist human activities in daily environments, such as offices, homes and hospitals, is growing rapidly. Such robots must communicate with humans and accomplish tasks in the human's daily environments. Our daily environments are constructed so as to accommodate human bodies. Therefore, a humanoid with the anthropoid shape (two arms and two legs) is a uniquely appropriate form.

Since a biped humanoid inherently suffers from instability and always risks to tipping over, stable and reliable biped walking is the most important goal. To reach this goal, many problems must be solved in the software as well as in the hardware. One way is to develop a software tool that allows us to evaluate both the software and the hardware.

It is useful and necessary to equip the software with control algorithms for the humanoid. Since the humanoid may tip over during walking, actual humanoid experiments carrying developing control algorithms are dangerous and costly. It is effective and convenient to verify various control algorithms, using a software simulator. In hardware, the humanoid physical capability is closely related to its components such as actuators and structure materials. For example, a humanoid robot with low power actuators and compliant materials can only walk at low speed. High speed walking requires high power actuators and rigid materials. Hence, to develop an hardware humanoid with the desired physical capability, it is desirable to clarify the required specifications of components in advance. A humanoid simulator, consisting of component parameters, is a necessary tool to solve this problem.

Many researchers have studied the humanoid, where main concerns were on balance control [1-10], actuator design [11-12] and artificial intelligence [13]. Although some researchers discussed the simulation environment for the humanoid, they focused on the computation of kinematics, dynamics, and/or the computer graphics [14-16].

The focus of this paper is on a humanoid simulator that can predict and show the same motion as a physical humanoid, including dynamics, geometry, actuators, sensors, controller and environmental factors. The paper is organized as follows: Section 2 introduces the dynamic analysis software package DADS (Dynamic Analysis and Design System), and presents the methods for constructing simulator components such as actuators, sensors and ground reaction force. Section 3 proposes a balance controller consisting of an off-line walk-pattern planner and a real-time modification control. Finally, simulation examples are provided in Section 4, followed by the conclusions in Section 5.

2 Simulator Construction

2.1 Simulation Environment

To predict the physical humanoid motion, it is necessary to accurately formulate and solve the kinematic and dynamic equations of the mechanisms. The humanoid is a system with multiple bodies and many degrees of freedom. The analytical complexity of nonlinear algebraic equations of kinematics and nonlinear differential equations of dynamics makes it practically impossible to obtain closed-form solutions. On the other hand, we can take the advantage of commercial dynamic software packages such as ADAMS, 3D Working Model, DADS and so on. The usefulness of such software packages has been confirmed in many areas. These software packages can automatically formulate the equations of kinematics and dynamics, solve the nonlinear equations, and provide computer graphics output of the simulation results [17].

After comparing the results of commercially available simulation software, we chose DADS due to its high numerical accuracy. Another benefit of DADS is that it incorporates easily with some popular CAD software packages such as CATIA, Pro/Engineer, and with control software packages such as MATLAB and control design tools. In addition, DADS accepts user-defined subroutines. These user-defined subroutines allow direct programmings in DADS. It is therefore possible to model detailed functions, define special constraints, and develop original control algorithms.

2.2 Simulator Components

The humanoid consists of a body, legs, and arms (Fig. 1). Each leg consists of thigh, shank and foot and has 6 degrees of freedom: 3 degrees of freedom in the



Fig. 1: A humanoid in simulator

hip joint, 1 in the knee joint, and 2 in the ankle joint. Each arm consists of upper arm, forearm, and hand, and has 7 degrees of freedom: 3 degrees of freedom in the shoulder joint, 1 in the elbow joint, and 3 in the wrist joint.

Model parameters such as link shape, size, mass and inertia can be arbitrarily designated in the simulator (For example, the measured human data or the actual humanoid parameters). Some components such as links, reduction devices (gears or pulley-belts), and mechanical ports can be modelled easily in DADS. In the following sections, we only discuss the key components such as actuators, interactions between feet and ground, sensors and balance controllers.

2.2.1 Actuator Model

Since actuators drive the humanoid, the humanoid physical capability is mainly decided by actuator performances. For example, low power actuators result in slow speed motion. If walking at excessively high speed is demanded or there are sudden disturbances, the robot cannot hold its stable posture and tips over easily.

The main performances of actuators can be denoted by power, torque, and an electrical time-constant. Let P_{max} be the maximal power, τ_{max} be the maximal torque, and T_e be the electrical time-constant. The following actuation constraints must be satisfied:

$$\tau = K_t e^{-\frac{1}{T_e}t} \tag{1}$$

$$\tau\nu \le P_{max} \tag{2}$$

$$\tau \le \tau_{max} \tag{3}$$

where τ and ν are the torque and the velocity, e is the voltage added on the motor, t is the time.

Constraints (1), (2) and (3) are considered in the simulator actuator model as shown in Fig. 2. When the maximal power, the maximal torque and the electrical time-constant of the used actuators are put in the simulator, we can obtain humanoid motion satisfying actuation constraints. By investigating which level of motion the humanoid can execute, it is possible to predict the humanoid ability based on its ac-



Fig. 2: Actuator simulator model

tuators. On the other hand, the minimum required specifications of actuators can be clarified by adjusting the values of parameters P_{max} , T_{max} and T_e , given the desired motion.

2.2.2 Contact Model

The reaction force between the feet and the ground strongly affects the behavior of the humanoid. For example, if the ground reaction force changes frequently, the humanoid will easily vibrate and become unstable. If the ground reaction force is too large, the burden on joints of supporting the robot becomes large; consequently the required torque and the power to drive joints increase. An accurate contact model is a key element for predicting the humanoid motion.

A spring-damper element is usually used to model the normal force between the feet and the ground. However the spring-damper model has some deficiencies [18-19]; for example, the ground normal force may be negative. A negative ground normal force implies that the ground pulls the contact foot, which is not physically correct for most ground. In order to obtain a suitable model of the ground reaction force, the Young's modulus-coefficient of restitution element was used [20].

Let E_m be the Young's modulus of the contacting material, and C_r be the coefficient of restitution. If $C_r = 0$, the contact is perfectly inelastic; if $C_r = 1$, the contact is perfectly elastic. The ground normal force is given by:

$$F_{n} = \begin{cases} 0.733 E_{m} \sqrt{\frac{1}{r}} \left[1 + \left(\frac{1 - C_{r}^{2}}{1 + C_{r}^{2}}\right) \tanh\left(2.5\frac{v_{p}}{v_{r}}\right) \right] \delta^{1.5} \\ |v_{p}| < |v_{t}| \\ 0.733 E_{m} \sqrt{\frac{1}{r}} \left[1 \pm 0.99 \left(\frac{1 - C_{r}^{2}}{1 + C_{r}^{2}}\right) \right] \delta^{1.5} \\ |v_{p}| = \pm |v_{t}| \\ 0.733 E_{m} \sqrt{\frac{1}{r}} \left[1 + \left(\frac{1 - C_{r}^{2}}{1 + C_{r}^{2}}\right) \right] \delta^{1.5} \\ |v_{p}| > |v_{t}| \\ |v_{p}| > |v_{t}| \end{cases}$$

$$(4)$$

where v_p and v_t denote the penetration velocity and the transition velocity between the contact foot and the ground, δ is the penetration, and r is the contact curvature. The friction force F_f is given by the following equation:

$$F_f = \mu F_n \tag{5}$$

where μ is the nominal friction coefficient.

2.2.3 Sensor Model

Sensors are necessary tools for the humanoid to know its state and the interaction between the humanoid and the environment. In the simulator, the rotary encoders in the joints, force/torque sensors at feet and inclination sensors on the body were considered. Since the values of joint angles and body inclinations can be obtained directly in DADS, we only discuss the model of the force/torque sensors.

The force/torque sensor is modeled by the simple beam theory [21]. As shown in Fig. 3, two rigid bodies are connected by a beam element. The force and torque is calculated according to the relative displacement and rotation between the two objects. Let $\mathbf{X} = [\Delta x, \Delta y, \Delta z, \Delta \theta_x, \Delta \theta_y, \Delta \theta_z]^T$ be the relative displacement and rotation in the x, y and z directions, the vector of force and torque $\mathbf{F} = [f_x, f_y, f_z, \tau_x, \tau_y, \tau_z]$ is computed as follows:

$$\mathbf{F} = \mathbf{K}\mathbf{X} + \mathbf{D}\dot{\mathbf{X}} \tag{6}$$

where $\dot{\mathbf{X}}$ is the differential of \mathbf{X} . \mathbf{K} is a 6×6 linear stiffness matrix, which is calculated according to the simple beam theory,

$$\begin{split} \mathbf{K} &= \\ \begin{bmatrix} \frac{E_b A}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12E_b I_x}{l^3} & 0 & 0 & 0 & \frac{-6E_b I_x}{l^2} \\ 0 & 0 & \frac{12E_b I_y}{l^3} & 0 & \frac{6E_b I_y}{l^2} & 0 \\ 0 & 0 & 0 & \frac{G_b I_x}{l} & 0 & 0 \\ 0 & 0 & \frac{6E_b I_y}{l^2} & 0 & \frac{4E_b I_y}{l} & 0 \\ 0 & \frac{-6E_b I_x}{l^2} & 0 & 0 & 0 & \frac{4E_b I_z}{l} \end{bmatrix}$$

where E_b is the Young's modulus, l is the length, G_b is the shear modulus, I_x , I_y and I_z are the area moments of inertia of the beam element in the x, y, z directions. **D** is a diagonal damping matrix:

$$\mathbf{D} = 2r_z \sqrt{\mathbf{M}_{\mathbf{diag}} \mathbf{K}_{\mathbf{diag}}} \tag{8}$$



Fig. 3: Force simulator model

Where r_z is the damping ratio, \mathbf{M}_{diag} is the diagonal mass matrix entries of either object 1 or object 2, whichever has the greater mass. \mathbf{K}_{diag} is the diagonal stiffness matrix of matrix (7). By adjusting the value of Young's modulus E_b , different stiffness force/torque sensors can be modeled.

3 Balance Controller

Stable and reliable biped walking has been the focus of biped humanoid research and has been studied mainly form two approaches. The first approach is to generate a stable walking pattern off-line [2-5], a method that assumes that the models of the robot and the environment are known. Since some properties such as uneven ground surface and environmental disturbances are difficult to know in advance, it is difficult to obtain an accurate model of the actual environment. Therefore, the second approach is the real-time control based on sensor information [5-10]. To be able to simulate both approaches, a simulator controller consisting of an off-line walk-pattern planner and a real-time modification control was constructed (Fig. 4). In this section, we propose such a balance control method.

To evaluate dynamic stability, we use the concept of ZMP. The ZMP [1] is defined as the point on the ground where the sum of all the moments of active forces equals zero. If the ZMP is within the contact polygon between the feet and the ground, the biped robot is stable. Hereafter, the contact polygon is called the stable region.

The walking pattern is planned based on the ZMP off-line. If the actual humanoid model and the environmental conditions are the same as those of the planned walking pattern, then the humanoid can walk stably only if following the planned walking pattern. If there are unexpected factors in the actual environment, the planned walking pattern is modified automatically by real-time control based on sensor feedback.



Fig. 4: Structure of balance controller

3.1 Off-line Walking Pattern

For a biped humanoid to be able to walk on various ground conditions such as level ground, rough terrain and obstacle-filled environments, the robot must be capable of various types of foot motion. For example, a biped robot should be able to lift its feet high enough to negotiate obstacles, or have support feet with suitable angles that can match the roughness of the terrain. After determining the foot motion, the robot's stability should be maintained by its body motion. We have proposed a method of planning such walking patterns [3], and briefly outline here.

To simplify the analysis, only sagittal plane walking will be discussed in the following. For a sagittal plane, let vector $\mathbf{X}_F = [x_f(t), z_f(t), \theta_f(t)]^T$ denote each foot trajectory; let vector $\mathbf{X}_B = [x_b(t), z_b(t), \theta_b(t)]$ denote the hip trajectory (Fig. 5). Supposing that the period necessary for one walking step is T_c , the kth walking step begins with the heel of the right foot leaving the ground at $t = kT_c$, and ends with the heel of the right foot touching the ground at $t = (k + 1)T_c$. The characteristic constraints of the right foot slope are given by the following equations:

$$\theta_{f}(t) = \begin{cases} q_{gs}(k) & t = kT_{c} \\ q_{b} & t = kT_{c} + T_{d} \\ q_{f} & t = (k+1)T_{c} \\ q_{ge}(k) & t = (k+1)T_{c} + T_{d} \end{cases}$$
(9)

where q_b and q_f be the designated slope angles of the right foot as it leaves and lands on the ground (Fig. 5), T_d denotes the interval of the double-support phase, and $q_{gs}(k)$ and $q_{ge}(k)$ are the angles of the ground surface under the support foot.

To generate a smooth trajectory, it is necessary that the second derivatives (accelerations) $\ddot{\theta}_f(t)$ be continuous at all t, including all breakpoints $t = kT_c, kT_c + T_d$, $(k + 1)T_c, (k + 1)T_c + T_d$. To solve for the foot tra-



Fig. 5: Walking parameters

jectory that satisfies constraint (9) and the constraints of second derivative continuity, 3rd order spline interpolation was used [22]. Similarly, $x_f(t)$ and $z_f(t)$ can be obtained. By setting the values of q_b , q_f , $q_{gs}(k)$, $q_{ge}(k)$, and other parameters such as clearance and stride length, different foot trajectories can be produced.

From the viewpoint of stability, it is desirable that the body posture $\theta_b(t)$ is constant when the robot has no waist joint; in particular, $\theta_b(t) = 0.5\pi$ [rad] on level ground. Body motion along the z-axis has little effect on the position of the ZMP. We can specify $z_b(t)$ to be constant, or to vary within a small range. To get high stability and smooth body motion along the x-axis, the following steps were considered:

- (1) Generate a series of smooth $x_b(t)$.
- (2) Determine the final $x_b(t)$ with a large stability margin

 $x_b(t)$ during a one-step cycle can be described by the double-support phase and the single-support phase functions. Letting x_{sd} and x_{ed} denote distances along the x-axis from the hip to the ankle of the support foot at the beginning and the end of the single-support phase, respectively (Fig. 5), the following equation can be obtained:

$$x_b(t) = \begin{cases} kD_s + x_{ed} & t = kT_c \\ (k+1)D_s - x_{sd} & t = kT_c + T_d \\ (k+1)D_s + x_ed & t = (k+1)T_c \end{cases}$$
(10)

By using 3rd order spline interpolation, trajectory of $x_b(t)$ that satisfies the constraint (10) and the constraint of second derivative continuity can be obtained. Hence, series of smooth $x_b(t)$ are obtained by setting different values of x_{sd} and x_{ed} within fixed ranges, in particular $0.0 < x_{sd} < 0.5D_s$, $0.0 < x_{ed} < 0.5D_s$. Then, the smooth trajectory $x_b(t)$ with the largest stability margin can be found by exhaustive search calculation.

3.2 Real-time Modification

Although the planned walking pattern satisfies the dynamic stability constraint, the humanoid may tip over when walking on unknown irregular rough terrain or if affected by an external force. When the humanoid begins to tip, the body slants. If the body posture is not recovered in time, the tipping moment becomes large and the slanting increases. During tipping, in general, only one edge of the support foot is in contact with the ground. In addition, to walk continuously, the swing foot should land on the ground on time; for example, the humanoid will tip forward if the swing foot lands on the ground too late. To adapt to unknown environmental factors, we proposed a real-time



Fig. 6: Body posture control and actual ZMP control

modification consisting of body posture control, actual ZMP control and landing time control based on sensor feedback.

To maintain the desired body posture, it is possible to control the hip, knee, and ankle joints. But, since the hip joints are near the body, the most effective way is to control the hip joints (Fig. 6).

Using a foot force/torque sensor, we can measure the actual ZMP. For a sagittal plane, generally only the toe or the heel is in contact with the ground during tipping. By the definition of ZMP, in these cases, the actual ZMP is on the toe or on the heel, that is, the actual ZMP is on the boundary of the stable region. To avoid having only one edge of the foot contacting the ground, it is necessary to have some stability margin; that is, there should be some distance between the actual ZMP and the boundary of the stable region. The ankle joints of the support feet are controlled to keep the actual ZMP within the stable region with a stability margin (Fig. 6).

To walk continuously, the transition of the swing foot to the support foot must occur at an appropriate time. To land on the ground on time as in the planned walking pattern, the foot position along vertical direction is controlled as follows: if the foot lands on the ground too fast, the robot lifts its foot in proportion to the reaction force; if the foot lands on the ground too late, the robot lowers its foot. A block diagram of



Fig. 7: Block diagram of balance control

100 50 Torque [Nm] 0 -50 -100 -150 3.42 4.32 Time [s] 5.22 (a) Hip joint torque 100 5.0 Torque [Nm] 0 -50 -100 -150 3.42 5.22 4.32 Time [s] (b) Knee joint torque 100 5 0 Torque [Nm] 0 - 5 0 -100 -150 3.42 4.32 Time [s] 5.22 (c) Ankle joint torque 6 4 Velocity [rad/s] 2 0 - 2 -4 - 6 3.42 5.22 4.32 Time [s] (e) Hip joint velocity 6 4 Velocity [rad/s] 2 0 -2 -4 -6 3.42 5.22 4.32 Time [s] (d) Knee joint velocity 6 4 Velocity [rad/s] 2 0 -2 -4 -6 3.42 5.22 4.32 Time [s] (f) Ankle joint velocity

Fig. 8: Required actuator specification

4 Simulation Examples

The humanoid simulator was constructed on an SGI workstation (Indigo2, MIPS R10000). Parameters of the humanoid robot were set according to an adult with a 70 [kg] weight and a 1.7 [m] height.

Fig. 8 shows the required actuator specifications for the case of 0.5 [m] stride length and 2.0 [km/h] walking speed. These data can be used as an approximate measure for actuator selection when designing an actual humanoid.

To test the simulator functions and to verify the effectiveness of our proposed method, we simulated walks in various conditions. Fig. 9 shows an example of walking on a rough terrain. Fig. 10 shows the walks in environments in the presence of disturbance. The disturbance is assumed to be the unexpected collision force between the pendulum and the humanoid. Different collision forces add on the humanoid with d-ifferent mass of the pendulum. Using the developed simulator, we can observe that the humanoid can continue walking when hit by a 3.5 [kg] pendulum, but tips over when hit by a 5.0 [kg] pendulum.



Fig. 9: Walking on rough terrain

5 Conclusions

The results of this paper are summarized as follows:

- A humanoid simulator including dynamics, geometry, actuators, sensors, controllers, and environmental factors was constructed based on DADS.
- (2) Using this simulator, we can clarify the required specifications of components, and predict the humanoid ability based on the given components.
- (3) A balance controller consisting of an off-line walkpattern planner and a real-time modification was proposed.
- (4) The functions of the developed simulator and the effectiveness of the proposed balance control were evaluated by walking on a rough terrain, a soft ground and an environment in the presence of disturbances.

the entire balance controller is shown in Fig. 7.





(b) humanoid hitted by 5.0[kg] pendulm

Fig. 10: Walking in an environment with disturbance

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