Artif Life Robotics (1999) 3:55–60

ORIGINAL ARTICLE

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A study on optimal motion of a biped locomotion machine

Received: August 7, 1998 / Accepted: September 7, 1998

Abstract In this paper we propose a calculation method for the optimal trajectory of a biped locomotion machine which is based on inverse kinematics and inverse dynamics. First, the trajectory of the waist is expressed by a Fourier series, where the bases are selected appropriately so that the periodic boundary conditions are strictly satisfied. A biped locomotion machine establishes optimal walking by using kicking forces to the ground at the moment of switching legs. In order to include the effects of the kicking forces, additional terms that indicate the impulsive forces at the moment of switching legs are included in the formulation. Then the angles of each joint are determined by inverse kinematics, and using inverse dynamics, the input torques of each joint are expressed in terms of Fourier coefficients. By defining the performance index as a quadratic form of the input torques, the motion planning problem is formulated as an optimization problem of the trajectory of the waist, whose parameters are Fourier coefficients of the trajectory of the waist. Using the successive quadratic programming (SQP) method, the optimal trajectory of a biped locomotion machine is obtained.

Key words Biped locomotion · Optimal motion · Inverse kinematics · Inverse dynamics

Introduction

In a previous paper,[1] we studied the motion control of a biped locomotion machine and proposed a hierarchical controller for the system. The upper controller is a motion planning system and generates an appropriate walking trajectory. The lower one is a motion controlling system, and controls each joint using feedback control and also controls gait parameters included in the trajectory.

This paper deals with the optimal motion planning of a biped locomotion machine in the motion planning system. In this paper, the optimal motion planning is formulated as an optimization problem of the trajectory of the waist.[2] From the view point of kinematics, the trajectory of a biped locomotion machine is periodic, and the trajectory needs to satisfy the boundary conditions strictly at the moment of switching legs. In order to derive such trajectory, many calculation times are required. To date, several optimization methods for the trajectories of nonlinear, multibody systems have been proposed.[3] In some research,[4,5] the input torques are expressed as Fourier series, and the motion planning problem is reduced to the optimization problem, where Fourier coefficients of the input torques become its optimization parameters. Even if we use this method, however, many calculation times are required in order to satisfy the constraints corresponding to the boundary conditions mentioned above.

In this paper, a calculation method for the optimal trajectory of a biped locomotion machine is proposed, which is based on inverse kinematics and inverse dynamics. First, the trajectory of the waist is expressed as a Fourier series, where the bases are selected appropriately so that the periodic boundary conditions are strictly satisfied. Then, the angles of each joint are determined by inverse kinematics and using inverse dynamics. The input torques of each joint are expressed in terms of Fourier coefficients. The performance index is defined as a quadratic form of the input torques, and then the Fourier coefficients of the trajectory of the waist become its optimization parameters. As a result, the motion planning problem becomes as optimization problem of the trajectory of the waist whose parameters are the Fourier coefficients of the trajectory of the waist.

A biped locomotion machine establishes optimal walking by using kicking forces to the ground at the moment of switching legs. In order to include the effects of these kicking

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This work was presented, in part, at the Third International Symposium on Artificial Life and Robotics, Oita, Japan, January 19–21, 1998

forces, many terms of a Fourier series are required, and this needs lengthy calculation. The kicking forces acting on the system causes discontinuities in the velocities of the motion. Considering this fact, in this paper, Fourier coefficients are chosen as to express the discontinuity of the velocity at the moment of switching legs. Then, using the successive quadratic programming (SQP) method, the optimal trajectory of a biped locomotion machine is obtained.

Equations of motion

We consider a biped locomotion machine composed of the main body and two legs, as shown in Fig. 1. Each leg is composed of two links connected to each other with a one DOF (degree of freedom) rotational joint, and connected to the main body with a one DOF rotational joint. The motion of the biped locomotion machine considered here is constrained in the sagittal plane. The legs are numbered 1 and 2, where leg 1 is considered as the supporting leg. The links of leg 1 are numbered 1 and 2 from the supporting point to the main body, and the links of leg 2 are numbered 4 and 5 from the main body to the end of the leg. The main body is link 3. The actuators are located at joints 2, 3, 4, and 5, but there is no actuator at the supporting point (joint 1). We define a set of unit vectors $\{\boldsymbol{a}_{01}, \boldsymbol{a}_{02}, \boldsymbol{a}_{03}\}$ fixed to an inertial space whose origin corresponds to the supporting point, where the direction of $\boldsymbol{a}_{01}$ corresponds to the sagittal direction, and the direction of $\boldsymbol{a}_{02}$ is vertically up. Similarly, a set of unit vectors $\{\boldsymbol{a}_{i1}, \boldsymbol{a}_{i2}, \boldsymbol{a}_{i3}\}$ is defined whose origin is on joint i, where $\boldsymbol{a}_{i1}$ is along the axis of link i, and the direction of $\boldsymbol{a}_{i3}$ coincides with the rotational axis of joint i.

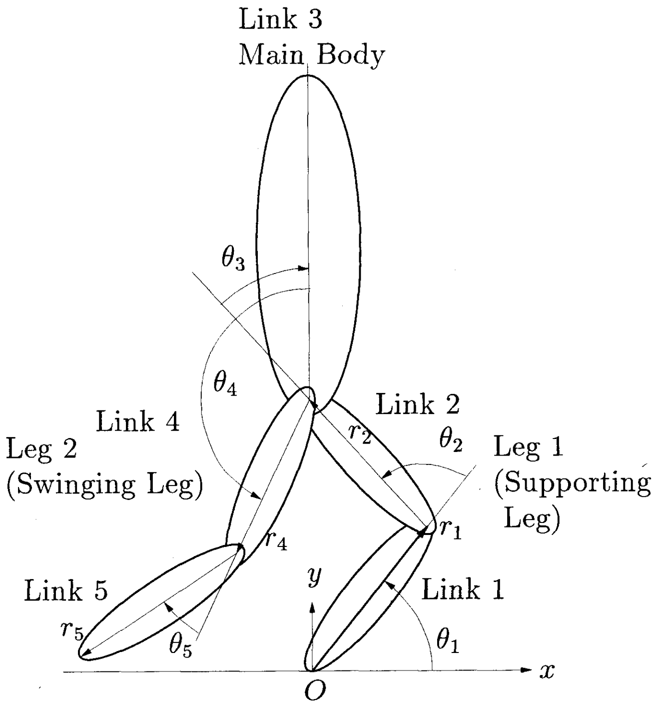

Fig. 1 Schematic model of biped locomotion

Using these unit vectors, we define the following unit column-matrices.

$$[\boldsymbol{a}_i]^T = [\boldsymbol{a}_{i1}\ \boldsymbol{a}_{i2}\ \boldsymbol{a}_{i3}] \qquad (i = 0, 1, 2, \ldots, 5)$$

We also define the following variables:
θ_i is the angle of rotation of link i relative to link $i-1$ about an axis of rotation of joint i.
A_{ij} is the transformation matrix from $[\boldsymbol{a}_j]$ to $[\boldsymbol{a}_i]$.

$$A_{ii-1} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boldsymbol{\omega}_{ij} = [\boldsymbol{a}_i]^T \omega_{ij}$ is the angular velocity vector of $[\boldsymbol{a}_i]$ to $[\boldsymbol{a}_j]$
$\boldsymbol{r}_i = [\boldsymbol{a}_i]^T r_i$ is the position vector from the origin of $[\boldsymbol{a}_{i-1}]$ to the origin of $[\boldsymbol{a}_i]$
$\boldsymbol{R}_i = [\boldsymbol{a}_i]^T R_i$ is the position vector from the origin of $[\boldsymbol{a}_i]$ to the center of mass of link i
$\boldsymbol{g} = [\boldsymbol{a}_0]^T g$ is the acceleration vector of gravity
We express a cross product of vector $\boldsymbol{x} = [\boldsymbol{a}]^T x$ as $\tilde{x}$ in $[\boldsymbol{a}]$ system as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \qquad \tilde{x} = \begin{bmatrix} 0 & x_3 & -x_2 \\ -x_3 & 0 & x_1 \\ x_2 & -x_1 & 0 \end{bmatrix} \tag{1}$$

We use the vector $\hat{\omega}$ as the state variable.

$$\hat{\omega} = \left[\hat{\omega}_{10}^T\ \hat{\omega}_{21}^T\ \hat{\omega}_{32}^T\ \hat{\omega}_{43}^T\ \hat{\omega}_{54}^T\right]^T$$
$$\hat{\omega}_{ij} = A_{i0}^T \omega_{ij} \qquad (i = 1, 2, \ldots, 5) \tag{2}$$

Generalized momenta relating to the state variable $\hat{\omega}$ are given as follows:

$$\hat{L} = \begin{bmatrix} \hat{L}_1 \\ \vdots \\ \hat{L}_5 \end{bmatrix} = K\hat{\omega} \tag{3}$$

$$K = \begin{bmatrix} K_1 \\ \vdots \\ K_5 \end{bmatrix} = H^T\left(\mathcal{L}^T M \mathcal{L} + J\right)H$$

$$\mathcal{L} = \mathcal{L}_r + \mathcal{L}_R$$

$$\mathcal{L}_r = \begin{bmatrix} 0 & & & & \\ \widetilde{A_{10}^T r_1} & & & O & \\ \widetilde{A_{10}^T r_1} & \widetilde{A_{21}^T r_2} & & & \\ \widetilde{A_{10}^T r_1} & \widetilde{A_{21}^T r_2} & & & \\ \widetilde{A_{10}^T r_1} & \widetilde{A_{21}^T r_2} & 0 & \widetilde{A_{43}^T r_4} & 0 \end{bmatrix}$$

$$\mathcal{L}_R = \begin{bmatrix} \widetilde{A_{10}^T R_1} & & & O \\ & \widetilde{A_{10}^T R_2} & & \\ O & & \ddots & \\ & & & \widetilde{A_{54}^T R_5} \end{bmatrix}$$

$$H = \begin{bmatrix} I_3 & & O \\ \vdots & \ddots & \\ I_3 & \dots & I_3 \end{bmatrix}$$

$$M = \begin{bmatrix} m_1 I_3 & & & \\ & m_2 I_3 & & O \\ O & & \ddots & \\ & & & m_5 I_3 \end{bmatrix}$$

$$J = \begin{bmatrix} J_1 & & & \\ & J_2 & & O \\ O & & \ddots & \\ & & & J_5 \end{bmatrix}$$

where, I_3 is 3×3 identity matrix and J_i $(i = 1, \dots, 5)$ are inertia matrices of each link at the center of mass. m_i $(i = 1, \dots, 5)$ are the masses of each link.

Equations of motion relating to generalized momenta $\hat{L}$ are derived as follows:

$$\dot{\hat{L}}_1 = \hat{G}_1 \tag{4}$$

$$\dot{\hat{L}}_2 + \tilde{V}_2^T \hat{P}_2 = \hat{G}_2 + \hat{u}_2 \tag{5}$$

$$\dot{\hat{L}}_3 + \tilde{V}_3^T \hat{P}_3 = \hat{G}_3 + \hat{u}_3 \tag{6}$$

$$\dot{\hat{L}}_4 + \tilde{V}_4^T \hat{P}_4 = \hat{G}_4 + \hat{u}_4 \tag{7}$$

$$\dot{\hat{L}}_5 + \tilde{V}_5^T \hat{P}_5 = \hat{G}_5 + \hat{u}_5 \tag{8}$$

where

$$\hat{P} = \begin{bmatrix} \hat{P}_1 \\ \vdots \\ \hat{P}_5 \end{bmatrix} = H^T P, \qquad P = \begin{bmatrix} P_1 \\ \vdots \\ P_5 \end{bmatrix} = M \mathcal{L} H \hat{\omega}$$

$$V = \begin{bmatrix} V_1 \\ \vdots \\ V_5 \end{bmatrix} = \mathcal{L}_r H \hat{\omega}$$

$$\hat{G} = \begin{bmatrix} \hat{G}_1 \\ \vdots \\ \hat{G}_5 \end{bmatrix} = H^T \mathcal{L}^T G, \qquad G = \begin{bmatrix} m_1 g & & \\ & \ddots & \\ & & m_5 g \end{bmatrix}$$

and u_i is a control torque acting at joint i.

As mentioned above, a biped locomotion machine establishes optimal walking by using kicking forces to the ground at the moment of switching legs. Here, the kicking forces are modeled as impulsive forces. The variances of angular momenta, $\hat{L}_i$, by the impulsive forces are denoted by $\Delta\hat{L}_i$. The variances $\Delta\hat{\omega}_{ij}$ of angular velocities $\hat{\omega}_{ij}$ are expressed in terms of $\Delta\hat{L}_i$ by integrating Eq. 3 over a small time interval $-\varepsilon \le t \le \varepsilon$, $\varepsilon > 0$ as

$$\sum_{i=1}^{5} K_{1i} \Delta\hat{\omega}_{ii-1} = \Delta\hat{L}_1 \tag{9}$$

$$\sum_{i=1}^{5} K_{2i} \Delta\hat{\omega}_{ii-1} = \Delta\hat{L}_2 \tag{10}$$

$$\sum_{i=1}^{5} K_{3i} \Delta\hat{\omega}_{ii-1} = \Delta\hat{L}_3 \tag{11}$$

$$\sum_{i=1}^{5} K_{4i} \Delta\hat{\omega}_{ii-1} = \Delta\hat{L}_4 \tag{12}$$

$$\sum_{i=1}^{5} K_{5i} \Delta\hat{\omega}_{ii-1} = \Delta\hat{L}_5 \tag{13}$$

The variance $\Delta\hat{\omega}_{32}$ of the angular velocity of the main body is expressed by using Eqs. 9–13 as follows:

$$\begin{aligned} \Delta\hat{\omega}_{32} = K_{13}^{-1}\big(&K_{11}\Delta\hat{\omega}_{10} + K_{21}\Delta\hat{\omega}_{21} \\ &+ K_{43}\Delta\hat{\omega}_{43} + K_{54}\Delta\hat{\omega}_{54}\big) \end{aligned} \tag{14}$$

Optimization of motion

In this section, we consider the optimization of the trajectory of the biped locomotion machine. Figure 2 shows the trajectory of a biped locomotion machine in the sagittal plane. The desired stride, S, and the desired walking period, t_f, are given. Then, the trajectory of the end of the swinging leg z_c is given as a function of the stride S, period t_f, and time t as follows:

$$\boldsymbol{z}_c = [\boldsymbol{a}_0]^T z_c \tag{15}$$

$$z_c = \begin{bmatrix} x_c \\ y_c \end{bmatrix} \tag{16}$$

$$x_c = x_c(t, S, t_f) \tag{17}$$

$$y_c = y_c(t, S, t_f) \tag{18}$$

The trajectory of the waist in the sagittal plane z_b is expressed by a Fourier series as follows:

$$\boldsymbol{z}_b = [\boldsymbol{a}_0]^T z_b \tag{19}$$

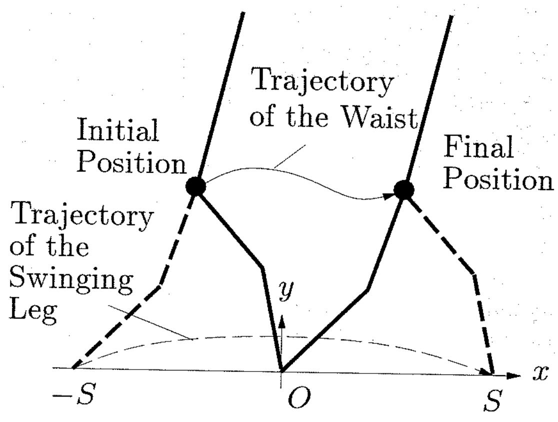

Fig. 2 Trajectory of a biped locomotion machine in the sagittal plane

$$z_b = \begin{bmatrix} x_b(t) \\ y_b(t) \end{bmatrix} \tag{20}$$

$$x_b(t) = vt + \alpha_0 + \sum_{n=1}^{N}\left\{\alpha_{3n-2}\sin\left(\frac{(2n-1)\pi}{t_f}t\right) + \alpha_{3n-1}\cos\left(\frac{2n\pi}{t_f}t\right) + \alpha_{3n}\sin\left(\frac{2n\pi}{t_f}t\right)\right\} \tag{21}$$

$$y_b(t) = \beta_0 + \sum_{n=1}^{N}\left\{\beta_{3n-2}\sin\left(\frac{(2n-1)\pi}{t_f}t\right) + \beta_{3n-1}\cos\left(\frac{2n\pi}{t_f}t\right) + \beta_{3n}\sin\left(\frac{2n\pi}{t_f}t\right)\right\} \tag{22}$$

$$v = \frac{S}{t_f} \tag{23}$$

Note that by using this expression for the trajectory, the continuity of the trajectory at the moment of switching legs is strictly satisfied. On the other hand, the effect of the impulsive force acting on the system is expressed as the discontinuity of the velocity at the moment of switching legs, which is expressed by the terms $\sin\frac{(2n-1)\pi}{t_f}t$.

Using inverse kinematics, the trajectories of each link of the legs are calculated as functions of the trajectory of the waist, z_b, and the trajectory of the end of the swinging leg, z_c, as follows:

$$\left.\begin{aligned} \theta_i &= \theta_i(t, x_b, y_b) \\ \omega_{ii-1} &= \omega_{ii-1}(t, x_b, y_b, \dot{x}_b, \dot{y}_b) \\ \dot{\omega}_{ii-1} &= \dot{\omega}_{ii-1}(t, x_b, y_b, \dot{x}_b, \dot{y}_b, \ddot{x}_b, \ddot{y}_b) \\ &\qquad (i = 1, 2) \end{aligned}\right\} \tag{24}$$

$$\left.\begin{aligned} \theta_4' &= \theta_4'(t, x_c, y_c) \\ \theta_5 &= \theta_5(t, x_c, y_c) \\ \dot{\theta}_4' &= \dot{\theta}_4'(t, x_c, y_c, \dot{x}_c, \dot{y}_c) \\ \omega_{54} &= \omega_{54}(t, x_c, y_c, \dot{x}_c, \dot{y}_c) \\ \ddot{\theta}_4' &= \ddot{\theta}_4'(t, x_c, y_c, \dot{x}_c, \dot{y}_c, \ddot{x}_c, \ddot{y}_c) \\ \dot{\omega}_{54} &= \dot{\omega}_{54}(t, x_c, y_c, \dot{x}_c, \dot{y}_c, \ddot{x}_c, \ddot{y}_c) \end{aligned}\right\} \tag{25}$$

where $\theta_4' = \theta_4 + \theta_3$ is the angle of rotation of link 4 relative to link 2.

The trajectories of the main body, θ_3, ω_{32}, are determined as follows: First, it is assumed that the inclination angle of the main body to the inertial space is small,

$$\theta_1 + \theta_2 + \theta_3 - \pi/2 \simeq 0 \tag{26}$$

Then Eq. 4 is linearized as

$$\begin{aligned} \dot{\omega}_{32} = F_1(\theta_i, \omega_{ii-1}, \dot{\omega}_{ii-1}\theta_4', \dot{\theta}_4', \ddot{\theta}_4')\theta_3 \\ + F_2(\theta_i, \omega_{ii-1}, \dot{\omega}_{ii-1}\theta_4', \dot{\theta}_4', \ddot{\theta}_4') \qquad (i = 1, 2, 5) \end{aligned} \tag{27}$$

In Eq. 27, variables θ_4', $\dot{\theta}_4'$, $\ddot{\theta}_4'$, θ_i, ω_{ii-1}, $\dot{\omega}_{ii-1}$ $(i = 1, 2, 5)$ are already calculated as functions of the trajectories of the waist and the end of the swinging leg in Eqs. 24 and 25, so Eq. 27 is a linearized differential equation for variable θ_{32} that determines the motion of the main body. In order to determine the trajectory of the main body, we have to solve Eq. 27 as a boundary value problem.

A boundary condition is given as follows

$$\theta_3(0) = \theta_3(t_f) + \{\theta_2(t_f) + \theta_1(t_f) - \theta_2(0) - \theta_1(0)\} \tag{28}$$

Equation 28 indicates that the inclination angles of the main body to the inertial space satisfy the continuity of the trajectory at the moment of switching legs. The trajectory of the main body is given as the solution of Eq. 27 under the boundary conditions of Eqs. 14 and 28.

Using inverse dynamics, the input torques of each joint during the single supporting phase are given as

$$\hat{u}_2(\theta_i, \omega_{ii-1}, \dot{\omega}_{ii-1}) = \dot{\hat{L}}_2 + \hat{V}_2^T\hat{P}_2 - \hat{G}_2 \tag{29}$$

$$\hat{u}_3(\theta_i, \omega_{ii-1}, \dot{\omega}_{ii-1}) = \dot{\hat{L}}_3 + \hat{V}_3^T\hat{P}_3 - \hat{G}_3 \tag{30}$$

$$\hat{u}_4(\theta_i, \omega_{ii-1}, \dot{\omega}_{ii-1}) = \dot{\hat{L}}_4 + \hat{V}_4^T\hat{P}_4 - \hat{G}_4 \tag{31}$$

$$\hat{u}_5(\theta_i, \omega_{ii-1}, \dot{\omega}_{ii-1}) = \dot{\hat{L}}_5 + \hat{V}_5^T\hat{P}_5 - \hat{G}_5 \qquad (i = 1, \ldots, 5) \tag{32}$$

The impulsive torques acting at each joint are expressed by integrating Eqs. 29–32 over a small interval $-\varepsilon \le t \le \varepsilon$, $\varepsilon > 0$ as follows:

$$\Delta\hat{u}_2 = \Delta\hat{L}_2 \tag{33}$$

$$\Delta\hat{u}_3 = \Delta\hat{L}_3 \tag{34}$$

$$\Delta\hat{u}_4 = \Delta\hat{L}_4 \tag{35}$$

$$\Delta\hat{u}_5 = \Delta\hat{L}_5 \tag{36}$$

The performance index is composed of two parts. The first part evaluates the cost of the input torque of each joint in the single supporting phase. The other one estimates the effects of impulsive forces at the moment of switching legs. The performance index is defined in quadratic form as

$$J = J_L + J_H \tag{37}$$

$$J_L = \int_0^{t_f}\sum_{i=2}^{5}\frac{R_i}{K_{Ti}^2}\hat{u}_i^2 dt \tag{38}$$

$$J_H = W\sum_{i=2}^{5}\hat{u}_i^2 \tag{39}$$

where W is a coefficient and R_i, K_{Ti} are values of the electrical resistor and torque coefficient of each actuator, respectively.

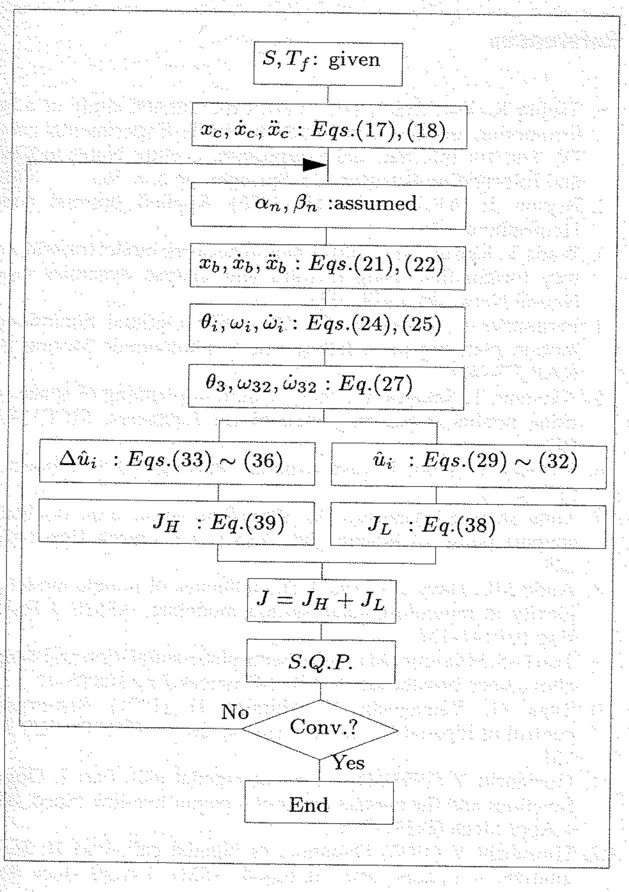

Fig. 3 Flow chart of motion planning (SQP: successive quadratic programming method)

Table 1 Physical parameters of the biped locomotion machine

	Length (m)	Mass (kg)	Inertia (kgm^2)
Links 1,5	0.20	1.5	5.0×10^{-3}
Links 2,4	0.20	2.0	7.0×10^{-3}
Link 3	0.30	5.0	1.0×10^{-2}

The first term of the performance index evaluates the energy consumption at the coils of electric DC actuators in the single supporting phase. The second term evaluates the impulsive one.

A motion planning algorithm of the biped locomotion machine proposed is shown in Fig. 3.

Numerical examples

Using the motion planning algorithm proposed, the dynamic performances of a biped locomotion machine are investigated. Table 1 shows the physical parameters of the biped locomotion machine used in the experimental setup. The trajectory of the waist is expressed using Fourier series up to the 2nd mode. Figures 4 and 5 show stick figures of a biped locomotion machine corresponding to the optimal solutions. Figure 4 shows the case without impulsive forces,

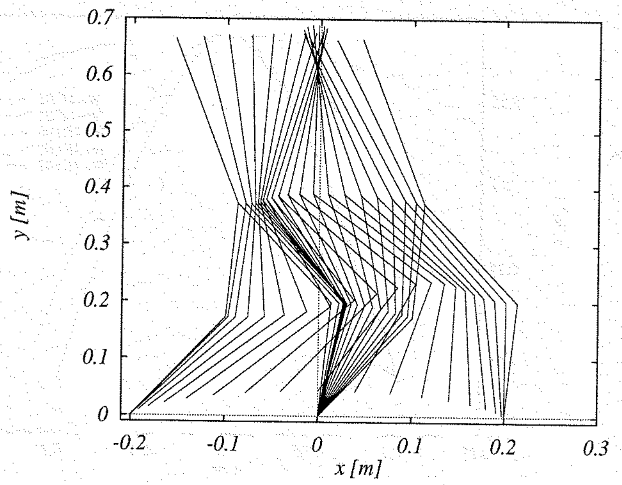

Fig. 4 Stick figure of a biped locomotion machine (without impulsive force)

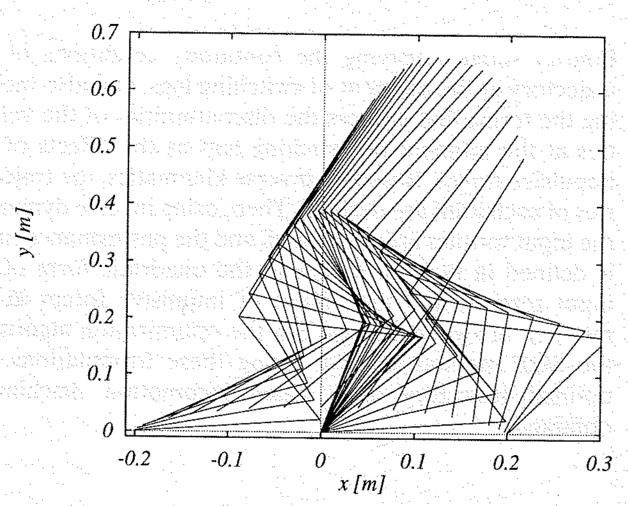

Fig. 5 Stick figure of a biped locomotion machine (with impulsive force)

and Fig. 5 shows that with impulsive forces. Figure 6 shows the trajectory of the waist with the weight value of the performance index W as a parameter. From these figures, the optimal trajectory of a biped locomotion machine is planned using the algorithm proposed in this paper, and we note that a biped locomotion machine has different walking patterns according to whether it uses the impulsive forces actively or not. When W is large, the center of mass of the system in the optimal motion derived is almost constant in height. On the other hand, when W is small, the optimal motion of the system derived becomes like an inverted pendulum.

Conclusion

A motion planning method for a biped locomotion machine is proposed. The trajectory of the waist is expressed by a

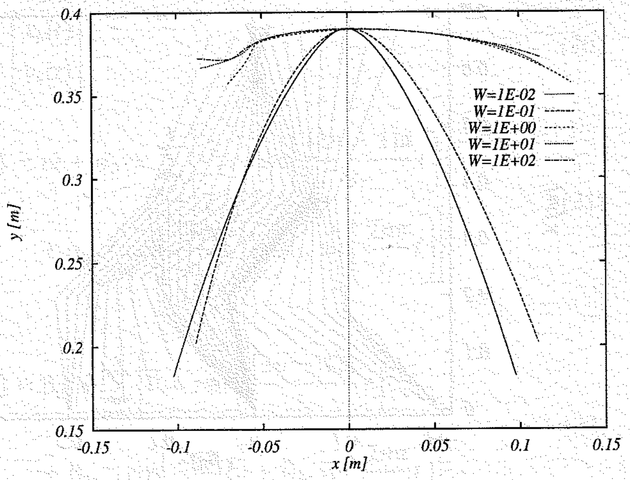

Fig. 6 Trajectory of the waist (*W* is a parameter)

Fourier series satisfying the continuity conditions of the trajectory at the moment of switching legs, and also including the terms that express the discontinuities of the velocities at the moment of switching legs as the effects of the impulsive forces. Based on inverse kinematics, the trajectories of each joint are derived. Then, using inverse dynamics, the input torques are calculated, and the performance index is defined in two parts, that is, the quadratic form of the input torques and the effects of impulsive forces at the moment of switching legs. For the optimization algorithm, the SQP method is used. Using these formulations, the optimal trajectory of a biped locomotion machine is obtained.

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