The Nonholonomic Manipulator
-Motion Planning and Feedback Control towards Practical Applications-

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Abstract

It is shown how a robot manipulator with \( n \) revolute joints can be controlled using only two actuators and passive mechanical transmissions. Exploiting unique features of nonholonomic systems, the nonholonomic manipulator was designed. The nonlinear kinematic model satisfies the chained form convertibility, which makes control easier.

In this paper, an efficient motion planning scheme for the nonholonomic manipulator is presented to minimize the kinematic overshoot. In addition, a feedback control, which is simple and easy to be implemented, is developed to achieve the control simplicity in practical applications.

1 Introduction

Mechanisms with nonholonomic constraints are often found controllable even though the number of actuators is less than that of generalized coordinates due to its nonlinearity. Exploiting this property, we proposed and fabricated the nonholonomic manipulator which is a controllable \( n \) joint manipulator with only two actuators [1, 2]. In order to create its nonholonomic constraint, a special type of velocity transmission, called the nonholonomic gear, was used.

The kinematic model of the nonholonomic manipulator can be converted to the chained form. Although there are many previously proposed controllers to the chained form, it is hard to say that we have efficient control schemes for the practical applications, i.e., sometimes it is too conservative and hard to be implemented, or algorithms are too complicated, which results in high computational cost.

In this paper, after the brief introduction of the nonholonomic manipulator, a motion planning and a feedback control scheme are presented, which satisfy control simplicity for practical applications and easy to be executed.

2 Design of the Nonholonomic Manipulator

2.1 The Nonholonomic Gear [1, 2]

In order to create nonholonomic constraints in manipulator design, the nonholonomic gear was used to transmit angular velocities. In figure 1, 3-dimensional view of the nonholonomic gear is presented. The basic components of this gear are a ball and several wheels which are input, output and supporting wheels. The velocity constraints of the ball are only due to point contacts with the wheels. Concerning the principle of the velocity transmission, see [1, 2]. The nonholonomic gear can be considered as a single input, multiple output CVT (continuously variable transmission), whose gear ratios are represented by trigonometric functions.

Figure 1: Nonholonomic gear with supporting wheels
2.2 Chained Form Conversion and a Prototype

Theoretical design of the nonholonomic manipulator was presented in [1]. Controllability can be shown by applying the Control Lie algebra. Although the manipulator is controllable, control problem is still not easy due to its nonlinearity. The kinematic model of the nonholonomic manipulator was designed so as to be converted to the chained form by nonlinear coordinate transformations and input transformations. Therefore, many previously proposed controllers can be applied for the control of the nonholonomic manipulator. The chained form considered here is given as follows [3]:

\[ \dot{z}_1 = v_1 \]  
\[ \dot{z}_2 = v_2 \]  
\[ \dot{z}_i = z_{i-1}v_1, \quad i \in \{3, \ldots, m\} \]  

We designed and constructed a four joint prototype nonholonomic manipulator as shown in figure 2.

3 Motion Planning

3.1 Kinematic Overshoot

Although nonholonomic chained systems are controllable, it cannot locally move to an arbitrary direction due to its constraints. For example, wheeled mobile robot cannot move sideways due to no-side-slip condition. This fact always results in an error between the shortest, or most preferred, path and a planned feasible path in the state space. We call this error the kinematic overshoot.

Notice that nonholonomic constraints of the chained systems are not constraints of generalized coordinates, but those of velocity constraints. Kinematic overshoot can be reduced using appropriate motion planning schemes. Minimizing the kinematic overshoot implies to approximate the holonomic path with the feasible nonholonomic path. In what follows, we approach this problem from parametric optimization that is more efficient in computation than the extensive optimization. We can plan the practical motion for the nonholonomic manipulator with feasible computational cost.

We define the shortest path as the line connecting the initial and desired configuration in the state space. This fact implies the approximation of a holonomic path in the state space by a piecewise straight path. We define the index of the kinematic overshoot by the distance between the shortest path and the points on the planned path. Conceptual illustration of the kinematic overshoot in the state space is shown in figure 3.

3.2 Sinusoidal Inputs

One of the motion planning schemes to the chained form is sinusoidal inputs to the chained form [4]. According to the proposed scheme, two inputs to the chained form are given as follows.

\[ v_1 = d_0 + d_1 \sin(\omega t) + \cdots + d_j \sin(j \omega t) \]  
\[ v_2 = e_0 + e_1 \cos(\omega t) + \cdots + e_k \cos(k \omega t) \]  

Coefficients of eq.(4), (5) are set to be \( j \in \{0,1\}, k \in \{0,1, \ldots, m-2\} \) in the proposed scheme. If we are given boundary conditions for a \( m \) dimensional chained form system, coefficients of sinusoids are given by solving algebraic equations.

3.3 Overparameterized Sinusoids

We propose the overparameterization of eq.(4), (5) by adding high frequency terms as a device to reduce the kinematic overshoot. By overparameterization, we can modify the trajectory so as to have a minimum kinematic overshoot satisfying the boundary condition. We extend the coefficients of eq.(4), (5) to \( j \in \{0,1, \ldots, j_f\}, k \in \{0,1, \ldots, m-2, \ldots, k_f\} \), where
$j_f, k_f$ are positive integer. In such a case, given boundary conditions and finite time $T = \frac{2\pi}{\omega}$ for control, coefficients can be solved from algebraic equations.

d1 is a parameter in eq. (4). $d_0$ depends on the boundary condition of $z_1$. For the case of the nonholonomic manipulator, $z_1$ have free boundary condition as explained in [2]. In addition, $d_i(i \in \{2, \cdots, j_f\})$, $c_i(i \in \{m-1, \cdots, k_f\})$ are parameters, which are from overparameterization, can be used for minimizing the kinematic overshoot. Therefore, there exist $j_f + k_f - 1$ parameters for 3 joint nonholonomic manipulator. By tuning these parameters using the gradient method on the surface of the kinematic overshoot, kinematic overshoot can be minimized.

3.4 Computed Results

Although exploiting high frequency terms helps reducing the kinematic overshoot, there is a limitation of setting inputs because computation of $z$ becomes highly complicated. In this section, we set $j_f = k_f = 4$ in eq. (4), (5). The parameter tuning was carried out in the 7 dimensional parametric space. The initial configuration $\theta(0)$ and the desired configuration $\theta_d$ were set to be $\theta(0) = [-20, -20, -20]^T (deg)$, $\theta_d = [20, 20, 20]^T (deg)$. Optimal parameters obtained were $d_0 = -0.000493$, $d_1 = -1.497$, $d_2 = -0.00166$, $d_3 = -0.00255$, $d_4 = -0.00413$, $c_5 = -0.0207$, $c_1 = -0.0447$, $c_0$, $c_1$ and $c_2$ are determined with the boundary conditions. The resultant kinematic overshoot in the state space was 0.0427. The resultant kinematic overshoot not exploiting overparameterization was 0.0556. 20% of the kinematic overshoot was reduced using overparameterization in this example.

In figure 4, joint angle trajectories with the optimal sinusoidal inputs are presented. To compare the planned motion with the shortest path, two trajectories are plotted in the state space in figure 5. It is obvious that planned path approximates the given reference path within a small error range. Figure 6 shows the motion of a 3 joint planar manipulator.

By using the sinusoidal inputs, motion with the small kinematic overshoot can be planned easily under the given boundary conditions. In addition, overparameterization plays a dominant role to achieve a goal.

4 Feedback Control of a Pseudo - Linearized System

Feedback control would be significant to live with various uncertainties. We introduce the new control strategy that can be applied to the control of the nonholonomic manipulator.
If we set a constant \( a_0 \) to the input \( v_1 \) of the chained form of Eq. (1) \(-\) (3), then the system is described as follows:

\[
\begin{align*}
\dot{z}_1(t) &= a_0 \\
\dot{z}_2(t) & \quad 0 \\
\vdots & \quad \\
\dot{z}_n(t) & \quad \end{align*}
\]

\[
A_{i,j} = \begin{cases} 
0 & i-j \neq 1 \\
a_0 & i-j = 1 
\end{cases}
\]

Equation (7) is a state equation of a linear system. We can apply the control scheme based on the linear system control theory. \( z_1 \) in Eq. (6) is controlled with open loop. For the case of the nonholonomic manipulator, we can set free boundary condition to \( z_1 \), which is a internal parameter. Therefore, by setting the proper \( a_0 \), \( z_1 \) goes to 0 with linear convergence. This determines a finite time to control.

When we stabilize the joint angles to the arbitrary values, the goal should be an equilibrium point. Equation (7) has its equilibrium point at \( z_i = 0 \), where \( i \in \{2, 3, \ldots, n\} \), and the arbitrary change of its location is not easy with the standard linear control theory. However, we can use the nonlinear coordinate transformation in [5] to stabilize the chained system to the arbitrary point. This coordinate transformation is valid for any control strategy to control the chained system to the origin. The linearized system of Eq. (7) can be stabilized to the origin easily by a state feedback scheme.

In Figure 7, time versus joint angles resulted from the feedback control scheme of a pseudo-linearized system, are plotted. The initial configuration \( \theta(0) \) and the desired configuration \( \theta \) were set to be \( \theta(0) = [10, 10, 10]^T\) (deg). \( \theta \) was \( [-10, -10, -10]^T\) (deg). \( \varepsilon_1 \) was 0.3272 (rad/sec) and finite time of convergence was 20 seconds. Feedback gains were \( [g_1, g_2, g_3]^T = [3.7131, -5.7865, 3.1623]^T \). We see that the joint angles reach the desired values after the finite time. Since the control scheme introduced here requires only simple computation, it can be used for the nonholonomic manipulator with greater number of joints.

5 Conclusions

Application of nonlinear control theory led us into developing a new innovative mechanism that never existed. We showed design of the nonholonomic manipulator and discussed efficient control schemes which satisfies control simplicity for practical applications. The

validity of the nonholonomic manipulator and the proposed control schemes were verified by presented simulation results. In practice, experiments were carried out successfully with the fabricated prototype.

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References


